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A USER'S MANUAL FOR THREE-DIMENSIONAL
HEATED SURFACE DISCHARGE COMPUTATIONS

Keith D. Stolzenbach, et al

Massachusetts Institute of Technology
Cambridge, Massachusetts

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A USER'S MANUAL FOR THREE-DIMENSIONAL
HEATED SURFACE DISCHARGE COMPUTATIONS

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ABSTRACT

In February 1971, a report by K. D. Stolzenbach and Donald R. F. Harleman entitled "An Analytical and Experimental Investigation of Surface Discharges of Heated Water" was published. (Technical Report No. 135, Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Department of Civil Engineering, M.I.T.; also published as Water Pollution Control Research Series Report No. 16130 DJU 02/71 by the Water Quality Office, Environmental Protection Agency, Washington, D.C.) The report described above presented a literature review of previous analytical and experimental research on heated surface discharges, the development of a predictive theory for calculating three-dimensional temperature distributions in the near-field region and verification of the theory based on laboratory work at M.I.T. and elsewhere. The above report also contained a listing of the computer program as originally developed.

Subsequent work and experience in using the computer program for the calculation of temperature distributions have resulted in modifications and improvements in the program. In addition, a number of runs covering a wide range of input parameters have been carried out. These runs were done with two objectives: (1) to explore the limits of applicability of the theory and (2) to prepare design charts showing the important characteristics of the near field temperature distribution for heated surface discharges. These charts will enable the designer to make rapid estimates of surface isotherms and vertical thickness of surface jets.

This report presents a review of the theoretical background for the three-dimensional temperature prediction model, a detailed discussion of the revised computer program and a case study illustrating the procedure for optimizing the design of a surface discharge channel. The revised computer program flow chart, program listing and a sample of the input and output data are given in the appendices.

One difference between the revised computer program presented in this report and the original computer program of February 1971 deserves further comment. The original program contained the possibility of considering a sloping bottom in the receiving water. The sloping bottom, was assumed to extend downward

in a linear slope from the bottom of the discharge channel. In the revised computer program it is assumed that the bottom of the receiving water does not interfere with the development of the surface jet. Thus the revised program corresponds to $S_x = \infty$ in the original program. The reasons for eliminating the bottom slope effect in the revised program are two-fold: (1) the mathematical model, as originally constituted, did not adequately predict the point of separation of the heated jet from the bottom or lateral spreading when the jet is in contact with the bottom; (2) from the standpoint of environmental impact, it may be desirable to accept the depth of the receiving water as a constraint and to design the surface discharge channel to minimize interference of the heated jet with the bottom.

It is recognized that almost none of the operating power plants that employ surface discharge schemes have been designed to minimize bottom impact. This fact makes it difficult to compare field data from many existing plants with temperature predictions based on the mathematical model presented in this report.

Further analytical and experimental studies of bottom interference with heated surface jets are presently underway in the M.I.T. Parsons Laboratory and will be the subject of a future technical report.

Inquiries relating to the availability of the program source deck should be directed to Professor D.R.F. Harleman, Room 48-335, M.I.T., Cambridge, Massachusetts 02139.

TABLE OF CONTENTS

<u>ABSTRACT</u>	<u>Page</u> iii
<u>TABLE OF CONTENTS</u>	v
<u>LIST OF FIGURES</u>	vi
<u>I. INTRODUCTION</u>	1
1.1 Characteristics of Heated Discharges	1
1.2 Surface Discharges	3
<u>II. THE MATHEMATICAL MODEL</u>	7
2.1 Basic Assumptions	7
2.2 Discharge Structure	9
2.3 Integration of the Equations	16
2.4 Solution of the Equations	20
<u>III. THE PROGRAM</u>	39
3.1 Dimensionless Equations	39
3.2 The Computational Scheme	43
3.3 Input Formats	45
3.4 Output Format	46
<u>IV APPLICATION</u>	47
4.1 Schematization	47
4.2 Use of the Program Output	56
4.3 Case Study - Heated Surface Discharge into a Receiving Water Body of Finite Depth	57
<u>ACKNOWLEDGEMENT</u>	65
<u>REFERENCES</u>	67
<u>LIST OF SYMBOLS</u>	69
APPENDIX I. Flow Chart for Solution of $[a_{ij}] \frac{dy}{dx} = c_i$	73
APPENDIX II. Program Listing	75
APPENDIX III. Input and Output	97

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1.1	Schematic of Heated Discharge	4
2.1	Coordinate Definitions	8
2.2	Discharge Structure	11
2.3	Calculated Surface Discharge Parameters: $F_o = 4.4, A = 0.35, k/u_o = 4.2 \times 10^{-5}, V/u_o = 0$	22
2.4	Calculated Isotherms of $\Delta T/\Delta T_o$: $F_o = 4.4, A = 0.35, k/u_o = 4.2 \times 10^{-5}, V/u_o = 0$	23
2.5	Centerline Temperature Rise $\Delta T_c/\Delta T_o$ and Dilution, D , in the Stable Region for $k/u_o = V/u_o = 0$	26
2.6	Maximum Vertical Depth of the Jet, $\frac{(h+r)_{\max}}{\sqrt{h_o b_o}}$ for $k/u_o = V/u_o = 0$	27
2.7a	Jet Parameters for $F_o = 20.0, V/u_o = 0, k/u_o = 0$	29
2.7b	Jet Parameters for $F_o = 10.0, V/u_o = 0, k/u_o = 0$	30
2.7c	Jet Parameters for $F_o = 5.0, V/u_o = 0, k/u_o = 0$	31
2.7d	Jet Parameters for $F_o = 2.0, V/u_o = 0, k/u_o = 0$	32
2.7e	Jet Parameters for $F_o = 1.0, V/u_o = 0, k/u_o = 0$	33
2.7f	Jet Parameters for $F_o = 20.0, V/u_o = 0.025, k/u_o = 0$	34
2.7g	Jet Parameters for $F_o = 10.0, V/u_o = 0.05, k/u_o = 0$	35
2.7h	Jet Parameters for $F_o = 5.0, V/u_o = 0.1, k/u_o = 0$	36
2.7i	Jet Parameters for $F_o = 2.0, V/u_o = 0.1, k/u_o = 0$	37
4.1	Coefficient of Thermal Expansion for Fresh Water $\beta = \frac{1}{\rho} \frac{\partial \rho}{\partial T} \text{ (}^\circ\text{F}^{-1}\text{) as a Function of Temperature } T(^{\circ}\text{F)}$	49
4.2	Discharge Channel Schematization	51

<u>Figure</u>		<u>Page</u>
4.3	Limitations on Maximum Jet Thickness by Bottom Topography	52
4.4	Two Layer Flow in the Discharge Channel	53
4.5	Temperature Distribution Plotting Aid	58
4.6	Schematic of Case Study Problem	59
4.7	Calculated Contours for the Case Study Example	64

I. Introduction

1.1 Characteristics of Heated Discharges

The discharge of heated condenser water into natural bodies of water can be broadly classified into two groups: surface discharges and submerged discharges. The latter class includes single port submerged discharges and multi-port diffusers. From the standpoint of minimizing the environmental impact of thermal effluents, the surface discharge has a number of advantages when compared with submerged discharges: (1) By careful design of a surface discharge it is possible to avoid temperature rises and high velocities along the bottom of the receiving water. (2) The time of travel of organisms entrained at the condenser water intake is short. (3) Heat dissipation from the surface of the receiving water is high because of the tendency of the discharge to form a stratified surface layer.

A typical open cycle system withdraws water from a natural water body through an intake structure and passes the flow through the turbine condensers where it undergoes a temperature rise before being returned to the receiving water through a discharge structure. In-plant technological considerations dictate condenser temperature rises of from 10-30°F with correspondingly large cooling water flows depending upon the amount of rejected heat.

The use of natural water bodies and coastal waters for disposal of waste heat must take into account the effect upon the environment of the flows and temperature rises induced in the receiving water. There is general agreement that water temperature increases which approach the sub-lethal range of impaired biological activity should be avoided. Practically all regulatory agencies provide this type of protection through controls on both maximum temperatures and allowable temperature rises and by designating the size of mixing zones for various types of receiving waters. In addition to environmental considerations the intake structure must be located and designed to prevent significant recirculation of heated water. Separation of the intake and discharge or the use of selective withdrawal structures are the most common techniques.

The temperature distribution induced in the receiving water by a heated discharge is determined by the characteristics of the discharge structure and by the local ambient heat transfer processes. Close to the point of discharge the momentum of the discharged water creates jet-like mixing of the heated and ambient water. Within this "near field" region the temperature and velocity

of the discharge decrease because of dilution by entrained water. The magnitude and extent of the dilution is determined primarily by the nature of the initial discharge flow, its submergence, velocity, dimensions and temperature rise above ambient. Mixing increases with increasing discharge momentum and decreases with increasing temperature rise. The greater the submergence of the discharge below the water surface, the lower the temperature rise at the surface will be after mixing. Mixing may also be affected by the presence of physical obstructions which tend to block the supply of dilution water. Surface discharges entrain a flow at least equal to the discharge flow in the near field, often up to twenty times as much.

Beyond the near field mixing region the discharge velocity and turbulence level are of the order of ambient values. In this "far field" region further entrainment does not occur and the temperature distribution is determined by natural turbulent convection and diffusion. Ultimately all of the rejected heat contained in the discharge passes to the atmosphere through the water surface, a process driven by the elevated surface temperatures. These far field heat transfers are highly variable, being determined by local water currents, wind, and meteorological conditions.

Consideration of either environmental impact or recirculation on discharge design must start with the near field temperature distribution. Far field processes are generally an order of magnitude less efficient in reducing temperatures and the far field temperature distribution tends to be more dependent upon the total amount of waste heat rather than the discharge design. In contrast, a wide range of dilutions is achievable in the near field through diverse types of discharge structures. It has been common practice to analyze near field temperatures by constructing scale models of the discharge structure. There is a pressing need for analytical models which relate the discharge characteristics to the flow and temperature distribution in the receiving water. Furthermore, since the analysis must almost always be performed in advance of actual plant construction, the analytical model must be totally predictive, containing no undetermined phenomenological coefficients.

This report describes the basis, structure, and use of a predictive model of the three-dimensional behavior of surface discharges of heated water. Emphasis is placed upon the assumptions underlying the theory, the scope of its validity,

the nature of its limitations, and the proper application to actual discharges. For similar treatments of submerged discharges, the reader is referred to references 1 and 2.

1.2 Surface Discharges

The theory presented herein considers a discharge of heated water from a rectangular open channel at the surface of an ambient body of water of infinite extent in which a current may be flowing (see Figure 1.1). The three-dimensional temperature distribution depends upon the mixing between the discharged and ambient water and upon the rate of heat transfer to the atmosphere at the water surface.

Experimental investigations of three-dimensional buoyant surface jets have been reported by Tamai (3), Wiegel (4), Jen (5), Stefan (6) and Hayashi (7). Buoyant surface discharges are distinguished from non-buoyant turbulent jets by lateral gravitational spreading and by reduction of vertical entrainment such as described by Ellison and Turner (8) for the two-dimensional case. The net result of these two processes is that the velocity and temperature distributions are much wider than deep with the increased surface area raising the possibility (as suggested by Hayashi) for significant surface heat loss. Previous analytical treatments (Hoopes (9), Motz (10)) of heated surface discharges have failed to take into account, in a single three-dimensional theory, the roles of buoyancy, initial channel shape, turbulent entrainment, and surface heat loss upon the temperature distribution.

In this treatment the discharge is assumed to be a free turbulent jet with a well defined turbulent region in which velocity and temperature are related to centerline values by similarity functions. An unsheared core region is accounted for. Turbulent entrainment is represented by entrainment coefficients as first introduced by Morton, Taylor, and Turner (11) and applied to other buoyant jet problems by Morton (12) and Fan (1) among others. A major contribution of this work is the treatment of lateral buoyant spreading by incorporating an assumed distribution for the lateral velocity into the set of integrated governing equations. Surface heat loss is assumed to be determined by a single heat loss coefficient as defined by Edinger (13). In the presence of a cross current in the receiving water the jet is deflected by entrainment of ambient lateral momentum.

The basic philosophy behind the formulation of the theory is that the

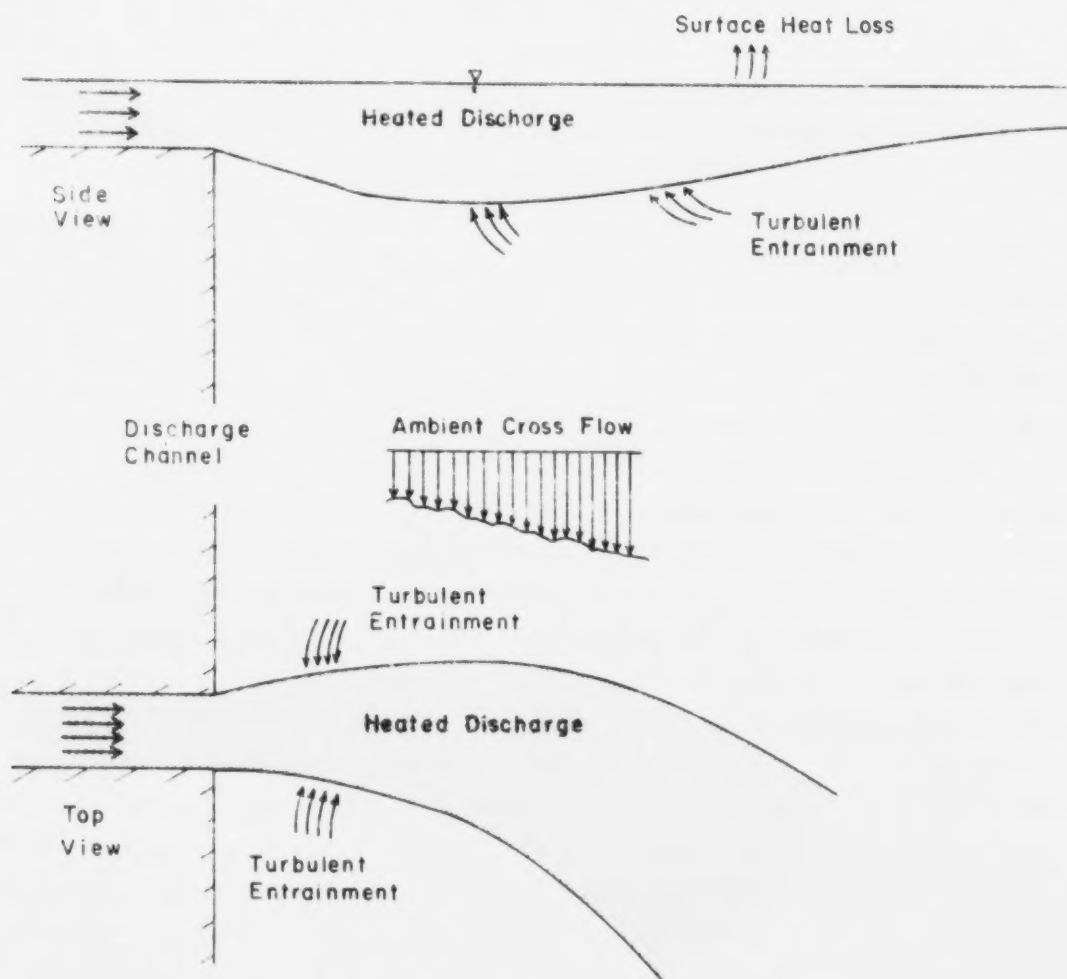


Fig. 1-1 Schematic of Heated Discharge

solution should match known non-buoyant jet behavior as the buoyancy terms go to zero. The effects of buoyancy are obtained by use of the basic equations and some judicious assumptions about the jet structure. In this way, the theory contains no undetermined coefficients and comparison of the theory with observations involves no curve fitting.

Because of the dependency of the treatment upon jet theory, the predictions of the model are valid only in the region where turbulence resulting from the discharge dominates ambient turbulent processes, i.e. in the near field.

II. The Mathematical Model

2.1 Basic Assumptions

The model considers a discharge Q_0 of heated water at temperature T_0 and density ρ_0 from a rectangular open channel of depth h_0 , width $2b_0$, and initial angle θ_0 at the surface of a receiving body of water at temperature T_a , density ρ_a and of large extent laterally and longitudinally. It is assumed that the bottom of the receiving water does not interfere with the vertical development of the surface jet. A non-uniform current V may be present in the receiving water. It is assumed that this current is parallel to the shoreline; however, its magnitude may vary in the offshore direction as shown in Figure 2.1. Far from the jet the water surface, η , is uniformly at $z = 0$. The flow in the receiving water is characterized by its velocity, density, pressure and temperature. These four variables are related by equations expressing conservation of mass, momentum and heat and an equation of state. The solution of the equations must be developed by setting certain terms in the basic equations equal to zero on the basis of the following assumptions:

- a) Steady flow: $\frac{\partial}{\partial t} = 0$
- b) Large Reynolds number: viscous terms negligible
- c) Boussinesq approximation: density gradients only important in pressure terms
- d) Hydrostatic pressure: $p = -\int_{\eta}^z \rho g dz$
- e) No jet induced motion at large depths: $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$ as $z \rightarrow -\infty$
- f) Boundary layer flow: $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$
- g) Small density differences: $\frac{\rho_a - \rho_0}{\rho_a} \ll 1$
- h) Mild jet curvature: $V/u_0 < 1$

With the above assumptions the basic equations of mass, momentum and heat conservation may be simplified to the following form:

Mass Conservation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2.1)$$

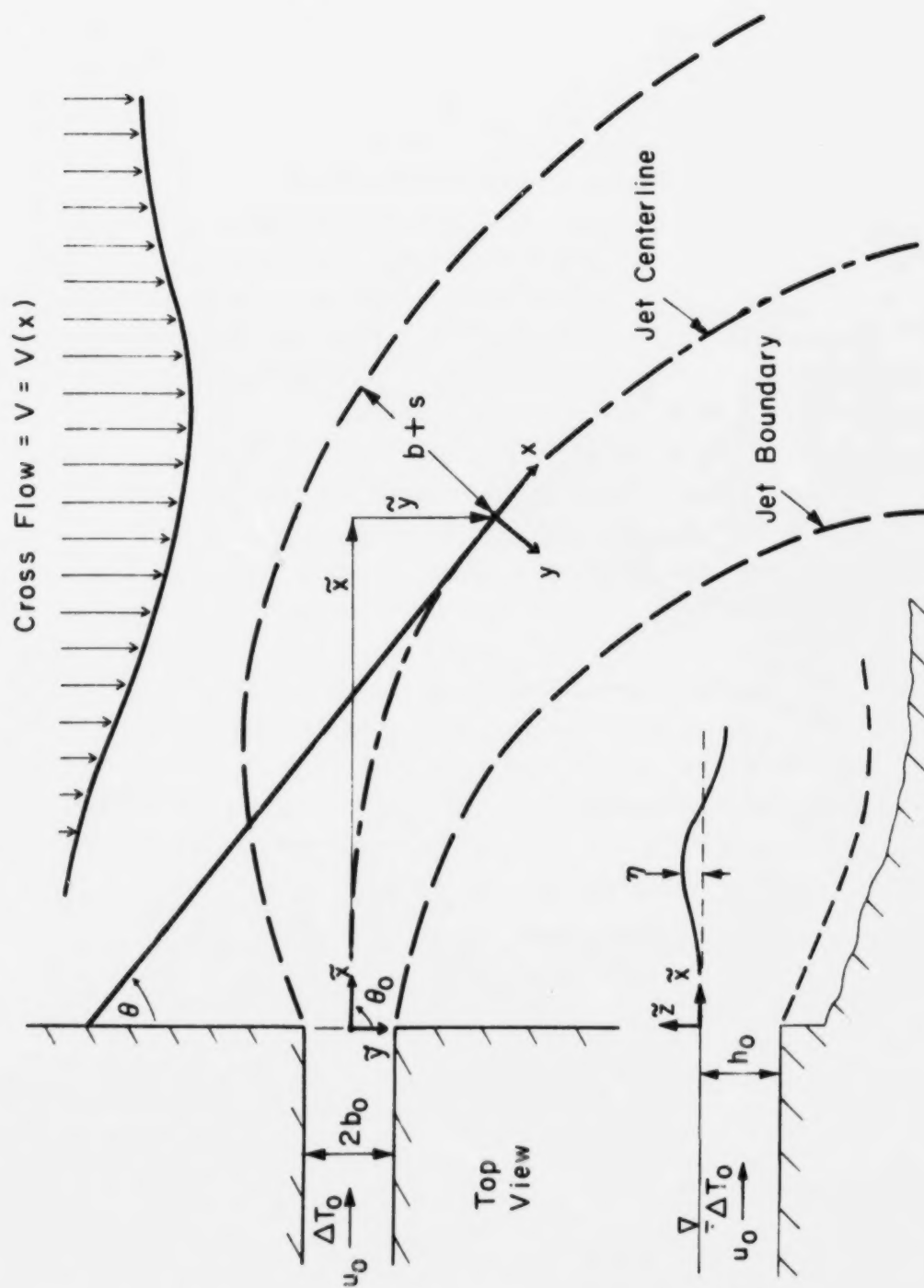


Fig. 2.1 Coordinate Definitions

x-Momentum Conservation

$$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = \beta g \int_z^\infty \frac{\partial T}{\partial x} dz - \frac{\partial u'v'}{\partial y} - \frac{\partial u'w'}{\partial z} \quad (2.2)$$

y-Momentum Conservation

$$\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = \beta g \int_z^\infty \frac{\partial T}{\partial y} dz + u^2 \frac{\partial \theta}{\partial x} \quad (2.3)$$

Heat Conservation

$$\frac{\partial uT}{\partial x} + \frac{\partial vT}{\partial y} + \frac{\partial wT}{\partial z} = - \frac{\partial v'T'}{\partial y} - \frac{\partial w'T'}{\partial z} \quad (2.4)$$

where:

x, y, z = coordinate directions relative to the jet centerline
(see Figure 2.1)

u, v, w = mean velocity components

u', v', w' = turbulent fluctuating velocity components

T = mean temperature

T' = turbulent fluctuating temperature

θ = centerline deflection angle (see Figure 2.1)

g = gravitational acceleration

$\beta = \frac{1}{\rho} \frac{\partial \rho}{\partial T}$ where ρ = density

2.2 Discharge Structure

The governing equations (2.1-2.4) assumed for heated surface discharges may not be solved without further manipulation since the turbulent transfer terms are not determined. The technique used to develop the solution is to assume a structure for the velocity and temperature within the discharge, and boundary conditions at the outer edges, leaving as unknowns only certain values such as the centerline velocity and temperature. The governing equations may then be integrated over a cross-section perpendicular to the discharge centerline. This procedure eliminates the unknown turbulent terms and yields a set of first order differential equations which may be solved for the variables describing the discharge behavior.

The assumed structure of the discharge is shown in Figure 2.2. The longitudinal velocity and temperature distributions are taken to be as follows where η is the water surface elevation and u_c and $\Delta T_c = T_c - T_a$ are the centerline velocity and temperature rise above ambient at $z = \eta$, $y = 0$:

$$\begin{aligned}
 u &= u_c + V \cos \theta & 0 < |y| < s & & -r < z < \eta \\
 \Delta T &= \Delta T_c \\
 \\
 u &= u_c f(\zeta_z) + V \cos \theta & 0 < |y| < s & & -(r+h) < z < -r \\
 \Delta T &= \Delta T_c t(\zeta_z) \\
 \\
 u &= u_c f(\zeta_y) + V \cos \theta & s < |y| < s+b & & -r < z < \eta \\
 \Delta T &= \Delta T_c t(\zeta_y) \\
 \\
 u &= u_c f(\zeta_y) f(\zeta_z) + V \cos \theta & s < |y| < s+b & & -(r+h) < z < -r \\
 \Delta T &= \Delta T_c t(\zeta_y) t(\zeta_z)
 \end{aligned}
 \tag{2.5}$$

where $\zeta_y = \frac{|y|-s}{b}$ and $\zeta_z = \frac{-z-r}{h}$

The lengths r and s pertain to the initial core region and h and b to the turbulent region of the jet (see Figure 2.2).

The particular forms of the similarity functions are assumed to be as follows:

$$\begin{aligned}
 f(\zeta) &= (1 - \zeta^{3/2})^2 \\
 t(\zeta) &= 1 - \zeta^{3/2}
 \end{aligned}
 \tag{2.6}$$

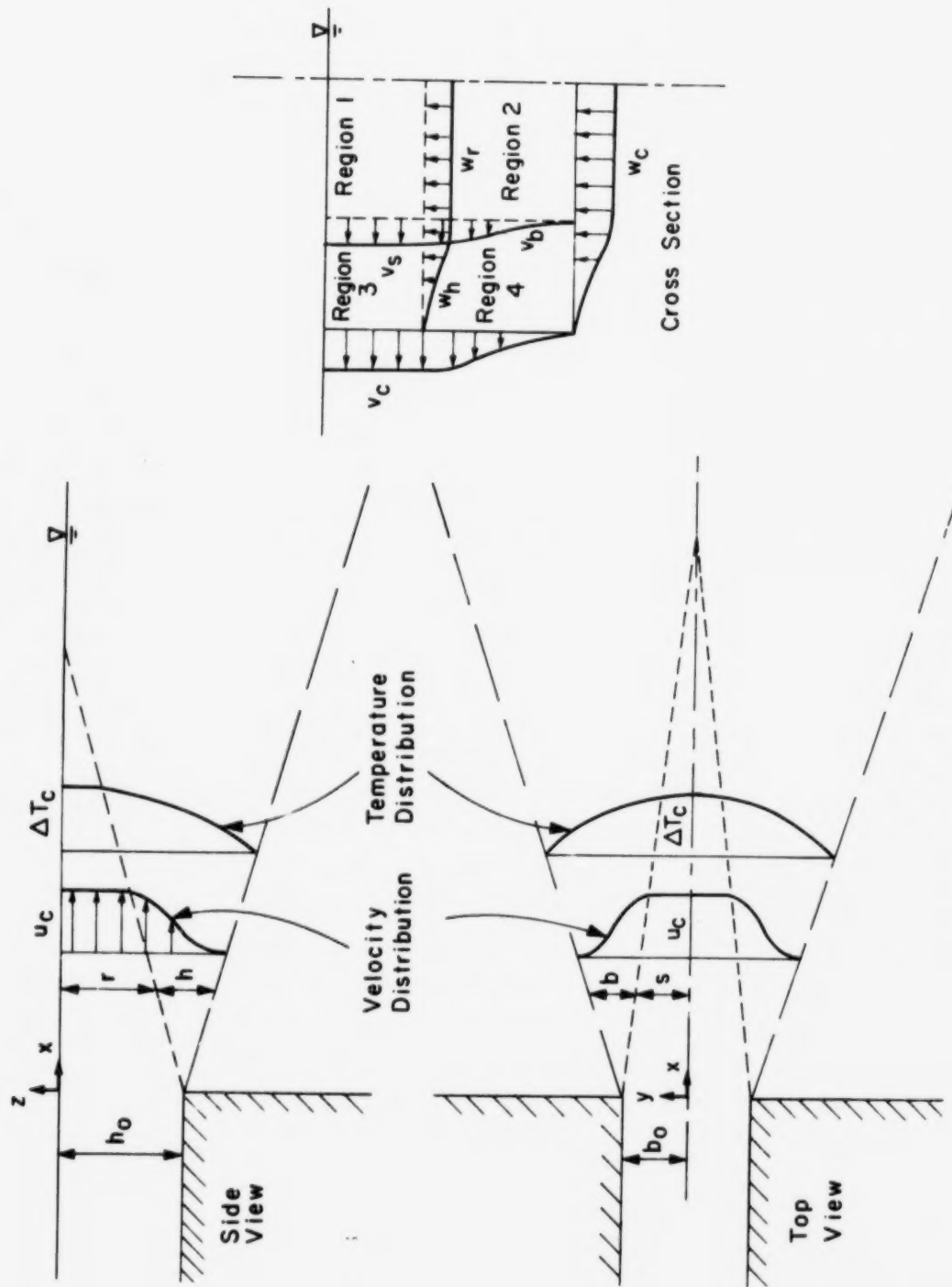


Fig. 2.2 Discharge Structure

The above functions were proposed by Abramovich (14). They have the desirable properties that a distinct jet boundary is defined at $\zeta = 1.0$ and that the velocity and temperature distributions are not identical but are related by $t = f^{1/2}$ as implied by Taylor's vorticity transfer theory.

The y momentum equation (2.3) expresses a balance between the lateral density gradients and lateral convective motions. It may be shown that for buoyant jets these lateral movements and thus the pressure gradient may be of the same order as convective transport in the x direction. The balance between lateral turbulent fluctuating pressures and Reynolds stresses, which is found in non-buoyant jets, is of second order and is neglected here. Since there are finite (but second order) lateral velocities in a non-buoyant jet, the lateral velocity v in equation (2.3) must be interpreted as the buoyant spreading velocity in excess of the non-buoyant value. To enable an integration of equation (2.3) over the jet cross section, the distribution of the lateral spreading velocity, v , must be specified. The lateral velocity is geometrically related to the longitudinal velocity by:

$$\frac{v}{u} = \tan \phi \quad (2.7)$$

where ϕ is the lateral jet streamline angle from the centerline in excess of the non-buoyant value. A distribution for u has already been given; it thus remains to choose a reasonable distribution for ϕ such that the following conditions are satisfied: (1) $\tan \phi = 0$ at $y = 0$ and (2) $\tan \phi = \left(\frac{db}{dx} - \epsilon \right)$ at $y = b+s$ where ϵ is the lateral spreading rate of a non-buoyant jet under the same conditions (cross flow, channel size, etc.). Since the gravitational spread is induced by the lateral temperature gradient, the y dependence of $\frac{\partial T}{\partial y}$ is used to distribute $\tan \phi$ between $y = 0$ and $|y| = b + s$. The result is:

$$\begin{aligned} v &= \pm \left(\frac{db}{dx} - \epsilon \right) u \zeta_y^{1/2} & s < |y| < s + b \text{ (v has the sign of y)} \\ v &= 0 & \text{elsewhere} \end{aligned} \quad (2.8)$$

The above distribution for v insures that for a non-buoyant jet in which $\frac{db}{dx} = \epsilon$ the gravitational spreading velocity is identically zero everywhere.

Depending upon whether r or s are non-zero at a given centerline distance, the jet cross section will have 1, 2, or 4 regions (see Figure 2.2) on each side of the centerline ($y = 0$). To permit integration of the governing equations over each region separately, the velocities and the turbulent transfer of heat and momentum are specified at the boundaries of the regions. At the centerline of the jet, symmetry implies that there is no net transfer of mass, momentum or heat:

$$v = u'v' = v'T' = 0 \quad y = 0 \quad (2.9)$$

At the boundaries between the regions the velocities are assumed to be:

$$\begin{aligned} u'w' &= 0 \\ \left. \begin{aligned} w &= w_r & 0 < |y| < s \\ w &= w_h f(\zeta_y) & s < |y| < s + b \\ w &= 0 & s + b < |y| \end{aligned} \right\} z = -r \\ & \\ u'v' &= 0 \\ \left. \begin{aligned} v &= \pm v_s & -r < z < \eta \quad (v \text{ has the sign of } y) \\ v &= \pm v_b f(\zeta_z) & -(r + h) < z < -r \\ v &= 0 & z < -(r + h) \end{aligned} \right\} |y| = s \end{aligned} \quad (2.10)$$

The internal velocities w_r , w_h , v_s , and v_b may be determined as part of the solution to the integrated governing equations but are of little interest in themselves.

The water surface is a boundary which permits no transfer of mass or momentum:

$$\left. \begin{aligned} u \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y} &= w \\ u'w' &= 0 \end{aligned} \right\} \quad z = \eta \quad (2.11)$$

The transfer of heat through the water surface is assumed to be proportional to the surface temperature rise above ambient:

$$w'T' = k(T - T_a) \quad z = \eta \quad (2.12)$$

The coefficient of heat loss, k , has units of velocity and is thus a kinematic quantity. It is related to the surface heat exchange coefficient, K , defined by Edinger (13) by:

$$k = \frac{K}{\rho c} \quad (2.13)$$

where ρ = density and c = specific heat of water. The determination of k (or K) for a particular case is discussed in a later section.

The outer boundaries of the jet is where entrainment of ambient water occurs but across which no heat is transferred. The boundary conditions are:

$$\begin{aligned} u'w' &= w'T' = 0 \\ w &= w_e - V \cos \theta \frac{dh}{dx} & 0 < |y| < s & \quad z = -(r+h) \\ w &= w_e f(\zeta_y) - V \cos \theta \frac{dh}{dx} & s < |y| < s+b & \\ & & & \quad (2.14) \\ u'v' &= v'T' = 0 \\ v &= \pm v_e + V \cos \theta \frac{db}{dx} & -r < z < \eta & \quad y = s+b \\ v &= \pm v_e f(\zeta_z) + V \cos \theta \frac{db}{dx} & -(r+h) < z < -r & \\ & & & \quad (v \text{ has the sign of } y) \end{aligned}$$

The velocities, w_e and v_e , are manifestations of the entrainment of ambient fluid into the turbulent region. In plane and axisymmetric non-buoyant jets, it is known that the entrainment velocity is proportional to the local center-line velocity. Note that Equation 2.8 gives $v = 0$ at $|y| = s + b$ where the boundary condition 2.14 specifies $|v| = v_e$. This is because the entrainment velocity v_e is an order of magnitude less than the spreading velocity v and its contribution to the integration of equation (2.3) is assumed to be balanced by the turbulent terms, which were neglected in the y equation. Ellison and Turner (8) have demonstrated that the vertical entrainment is a function of the gross Richardson number, $\frac{\beta g \Delta T_c h}{u_c^2}$, in two-dimensional jets. In this study

the entrainment velocities are assumed to be given by

$$\begin{aligned} \frac{v_e}{u_c} &= \alpha_y \\ \frac{w_e}{u_c} &= \alpha_z \exp \left\{ -C \frac{\beta g \Delta T_c h}{u_c^2} \right\} \end{aligned} \quad (2.15)$$

Ellison and Turner's data indicate a value of $C = 5.0$ as appropriate. The entrainment coefficients, α_y and α_z , are to be determined such that the solution for the non-buoyant case ($T_o = T_a$) agrees with the experimental observations that the growth of a non-buoyant turbulent region is symmetrical:

$$\begin{aligned} \frac{db}{dx} &= \frac{dh}{dx} = \epsilon \\ \frac{ds}{dx} &= \frac{dr}{dx} \end{aligned} \quad (2.16)$$

For non-buoyant jets discharging into a quiescent receiving water the spreading rate, ϵ_o , is constant. In these cases, Abramovich gives $\epsilon_o = .22$ for the similarity functions, f and t , used here.

The boundary condition at $x = 0$ is related to the discharge channel geometry, the flow rate Q_o , and the initial discharge temperature T_o .

$$\left. \begin{aligned} r &= h_o \\ s &= b_o \\ h &= b = 0 \\ u &= u_o = \frac{Q_o}{2h_o b_o} + V \cos \theta_o \\ \Delta T_c &= \Delta T_o \\ \theta &= \theta_o \\ \tilde{x} &= \tilde{y} = 0 \end{aligned} \right\} x = 0 \quad (2.17)$$

2.3 Integration of the Equations

With the velocity and temperature distributions and boundary conditions stated in the previous section, the equations of motion may be integrated over the y - z cross section of the jet. The x momentum and mass conservation equations are integrated over each of the four possible jet regions on each side of the centerline plane. (Integration over both sides of the jet results in redundant equations because of the assumed symmetry.) This yields eight equations. The y -momentum and heat equations are integrated over the entire half-jet cross-section, yielding two more equations. With this choice of integrating limits, the terms in the integrated y -equation represent a balance between the lateral gravitational force and the lateral spreading of the discharge. Another equation is generated from the y -equation by integrating it over the entire cross section of the jet on both sides of the centerline. In this case the lateral spreading terms all drop out, being anti-symmetrical, and the remaining terms give the rate of deflection of the jet in the presence of a cross current. The equation set, completed by a simple geometrical relationship between the coordinates (x,y) referred to the jet centerline and the fixed centerline coordinates $(\tilde{x},\tilde{y})$ is given in Table 2.1. Further details in the derivation of these equations are given in Stolzenbach and Harleman (15).

Region 1: continuity $rs \frac{d}{dx} [u_c + V \cos \theta] + rv_s - sw_r = 0$

Region 2: continuity $s \left[\frac{d}{dx} [h(u_c I_1 + V \cos \theta)] + (u_c + V \cos \theta) \frac{dr}{dx} + w_r - \alpha_{sz} u_c \right] + v_b h I_1 = 0$

Region 3: continuity $r \left[\frac{d}{dx} [b(u_c I_1 + V \cos \theta)] + (u_c + V \cos \theta) \frac{dr}{dx} - v_s + \alpha_{y} u_c \right] - w_r b I_1 = 0$

Region 4: continuity $\frac{d}{dx} \left[hb(u_c I_1^2 + V \cos \theta) \right] + (u_c I_1 + V \cos \theta) \left[b \frac{dr}{dx} + h \frac{ds}{dx} \right] + (w_h - \alpha_{sz} u_c) I_1 b$

$$- (v_b - \alpha_y u_c) I_1 h = 0$$

Region 1: x momentum $(u_c + V \cos \theta) \left[2rs \frac{d}{dx} [u_c + V \cos \theta] + rv_s - sw_r \right] + \beta g s \left[\frac{d}{dx} (\Delta T_c \frac{r^2}{2}) + I_3 r \frac{d}{dx} (\Delta T_c h) \right] = 0$

Region 2: x momentum $s \left[\frac{d}{dx} [h (u_c^2 I_2 + 2V \cos \theta u_c I_1 + V^2 \cos^2 \theta)] + [u_c + V \cos \theta]^2 \frac{dr}{dx} \right.$

$\left. + w_r [u_c + V \cos \theta] - \alpha_{sz} u_c V \cos \theta + \beta g \left[I_4 \frac{d \Delta T_c h^2}{dx} + I_3 \Delta T_c h \frac{dr}{dx} \right] + v_b h (u_c I_1 + V \cos \theta I_1) = 0 \right.$

Table 2.1 Integrated Equations for Deflected Buoyant Jets

$$\begin{aligned}
\text{Region 3: } x \text{ momentum} \quad & r \left[\frac{d}{dx} [b(u_c^2 I_2 + 2V \cos \theta u_c I_1 + V^2 \cos^2 \theta)] + [u_c + V \cos \theta]^2 \frac{ds}{dx} - \right. \\
& v_s [u_c + V \cos \theta] + \alpha_y u_c V \cos \theta \left. \right] - w_h^b [u_c I_2 + V \cos \theta I_1] + \beta g \left[\frac{I_3}{2} \frac{d \Delta T_c b r^2}{dx} \right. \\
& \left. + I_3^2 r \frac{d \Delta T_c b h}{dx} + \Delta T_c \left(\frac{r^2}{2} + I_3 h r \right) \frac{ds}{dx} \right] = 0
\end{aligned}$$

$$\begin{aligned}
\text{Region 4: } x \text{ momentum} \quad & \frac{d}{dx} [h b (u_c^2 I_2 + 2V \cos \theta u_c I_1^2 + V^2 \cos^2 \theta)] + [u_c^2 I_2 + 2V \cos \theta u_c I_1 + V^2 \cos^2 \theta] \\
& \left[b \frac{dr}{dx} + h \frac{ds}{dx} \right] + [w_h^b - v_b^b h] [u_c I_2 + V \cos \theta I_1] - u_c I_1 [\alpha_{sz} b - \alpha_y h] V \cos \theta \\
& + \beta g \left[\frac{I_3 I_4}{dx} \frac{d \Delta T_c h^2 b}{dx} + I_4 \Delta T_c h^2 \frac{ds}{dx} + I_3^2 \Delta T_c h b \frac{dr}{dx} \right] = 0 \\
\text{Jet } y \text{ momentum} \quad & \frac{d}{dx} \left[\frac{db}{dx} - \epsilon \right] [u_c^2 b I_6 (r + h I_2) + 2V \cos \theta u_c b I_5 (r + h I_1) + V^2 \cos^2 \theta b (r + h)] \\
& - \beta g \Delta T_c \left(\frac{r^2}{2} + I_3 r h + I_4 h^2 \right) = 0
\end{aligned}$$

Table 2.1 (cont'd) Integrated Equations for Deflected Buoyant Jets

$$\text{Jet Heat} \quad \frac{d}{dx} \left[u_c \Delta T_c (s + b I_7) (r + h I_7) + v \cos \theta \Delta T_c (s + b I_3) (r + h I_3) \right]$$

$$+ k \Delta T_c (s + b I_3) = 0$$

$$\text{Jet Bending} \quad \left[u_c^2 (s + b I_2) (r + h I_2) + 2 v \cos \theta u_c (s + b I_1) (r + h I_1) + \right.$$

$$\left. v^2 \cos^2 \theta (s + b) (r + h) \right] \frac{d\theta}{dx} - u_c v \sin \theta \left[-a_{sz} (s + b I_1) + a_y (r + h I_1) \right] = 0$$

$$\text{Jet x position} \quad \frac{dx}{dx} - \sin \theta = 0$$

$$\text{Jet y position} \quad \frac{dy}{dx} - \cos \theta = 0$$

$$I_1 = \int_0^1 f(\zeta) d\zeta = \int_0^1 (1 - \zeta^{3/2})^2 d\zeta = .4500 \quad I_4 = \int_0^1 \int_0^1 t(\zeta) d\zeta d\zeta = \int_0^1 \int_0^1 (1 - \zeta^{3/2}) d\zeta d\zeta = .2143$$

$$I_2 = \int_0^1 f^2(\zeta) d\zeta = \int_0^1 (1 - \zeta^{3/2})^4 d\zeta = .3160 \quad I_5 = \int_0^1 \int_0^1 f(\zeta) \zeta^{1/2} d\zeta = \int_0^1 \int_0^1 (1 - \zeta^{3/2})^2 \zeta^{1/2} d\zeta = .2222$$

$$I_3 = \int_0^1 t(\zeta) d\zeta = \int_0^1 (1 - \zeta^{3/2}) d\zeta = .6000 \quad I_6 = \int_0^1 \int_0^1 f^2(\zeta) \zeta^{1/2} d\zeta = \int_0^1 \int_0^1 (1 - \zeta^{3/2})^4 \zeta^{1/2} d\zeta = .1333$$

$$I_7 = \int_0^1 f(\zeta) t(\zeta) d\zeta = \int_0^1 (1 - \zeta^{3/2})^3 d\zeta = .3680$$

Table 2.1 (cont'd) Integrated Equations for Deflected Buoyant Jets

The values of the non-buoyant ($\Delta T_o = 0$) entrainment coefficients which satisfy equations 2.16 in addition to Table 2.1 are:

$$\begin{aligned}
 \alpha_y &= -(I_1 - I_2) \epsilon_o & \text{for } s > 0 \\
 \alpha_y &= -\frac{I_1 \epsilon_o}{2} & \text{for } s = 0 \\
 \alpha_z &= (I_1 - I_2) \epsilon_o & \text{for } r > 0 \\
 \alpha_z &= \frac{I_1 \epsilon_o}{2} & \text{for } r = 0
 \end{aligned} \tag{2.18}$$

where I_1 and I_2 are integration constants as given in Table 2.1.

2.4 Solution of the Equations

The thirteen equations in Table 2.1 are a first order system of dimensional differential equations in x for the variables: u_c , ΔT_c , h , b , r , s , θ , $\tilde{x}$, $\tilde{y}$, v_s , v_b , w_r , and w_h . The actual solution proceeds first by writing the equations in a dimensionless form by normalizing each variable by the characteristic values: u_o , $\Delta T_o = T_o - T_a$, and $\sqrt{h_o b_o}$. The solution is then determined by the following dimensionless parameters:

$$F_o = \frac{u_o}{\sqrt{\beta g \Delta T_o h_o}} = \text{initial densimetric Froude number}$$

$$A = h_o / b_o = \text{aspect ratio}$$

$$k/u_o = \text{heat loss parameter}$$

$$V/u_o = \text{cross flow parameter}$$

The computer program which solves the equations is described in Section IV along with instructions for its use. The output consists of values of u/u_o , $\Delta T_c / \Delta T_o$, $h/\sqrt{h_o b_o}$, $b/\sqrt{h_o b_o}$, $r/\sqrt{h_o b_o}$, $s/\sqrt{h_o b_o}$, θ , $\tilde{x}/\sqrt{h_o b_o}$ & $\tilde{y}/\sqrt{h_o b_o}$ as a function of $x/\sqrt{h_o b_o}$. In addition to these variables which describe the jet structure (u_c , ΔT_c , . . . , etc.), other interesting dependent quantities which are functions of x may be defined. In the buoyant jet the vertical entrainment is a function of the local densimetric Froude number (an inverse Richardson number), F_L :

$$F_L = \frac{u_c}{\sqrt{\beta g \Delta T_c h}} \quad (2.19)$$

where u_c , ΔT_c , and h are local values at a given distance from the origin. The total flow in the jet may be determined by integrating the x velocity, u , over the jet cross section. The ratio of the flow at a given x to the initial flow is the jet dilution, D , or Q as labeled by the output.

$$D = \frac{u_c (r + I_1 h)(s + I_1 b) + V \cos \theta (r + h)(s + b)}{(u_o + V \cos \theta_o) h_o b_o} \quad (2.20)$$

Similarly the effect of surface heat loss may be evaluated by calculating the ratio of convected excess heat flow in the jet to the initial excess heat flow, HT .

$$HT = \frac{u_c \Delta T_c (r + I_1 h)(s + I_1 b) + V \cos \theta \Delta T_c (r + h)(s + b)}{\Delta T_o (u_o + V \cos \theta_o) h_o b_o} \quad (2.21)$$

Finally a dimensionless time of travel along the jet centerline from the end of the discharge channel to x is computed.

$$TM = \frac{u_o}{\sqrt{h_o b_o}} \int_0^x \frac{dx}{u_c} \quad (2.22)$$

The structure of a heated surface discharge is shown for a particular theoretical calculation in Figures 2.3 and 2.4. The main features are:

- 1) A core region in which the centerline velocity is constant and the centerline temperature rise decreases very slightly. The dilution, D , and the local densimetric Froude number, F_L , do not vary greatly in this region. The magnitude of F_L is much larger than F_o because the initial depth of the turbulent region, h , is zero. There is no significant surface heat loss in the core region.

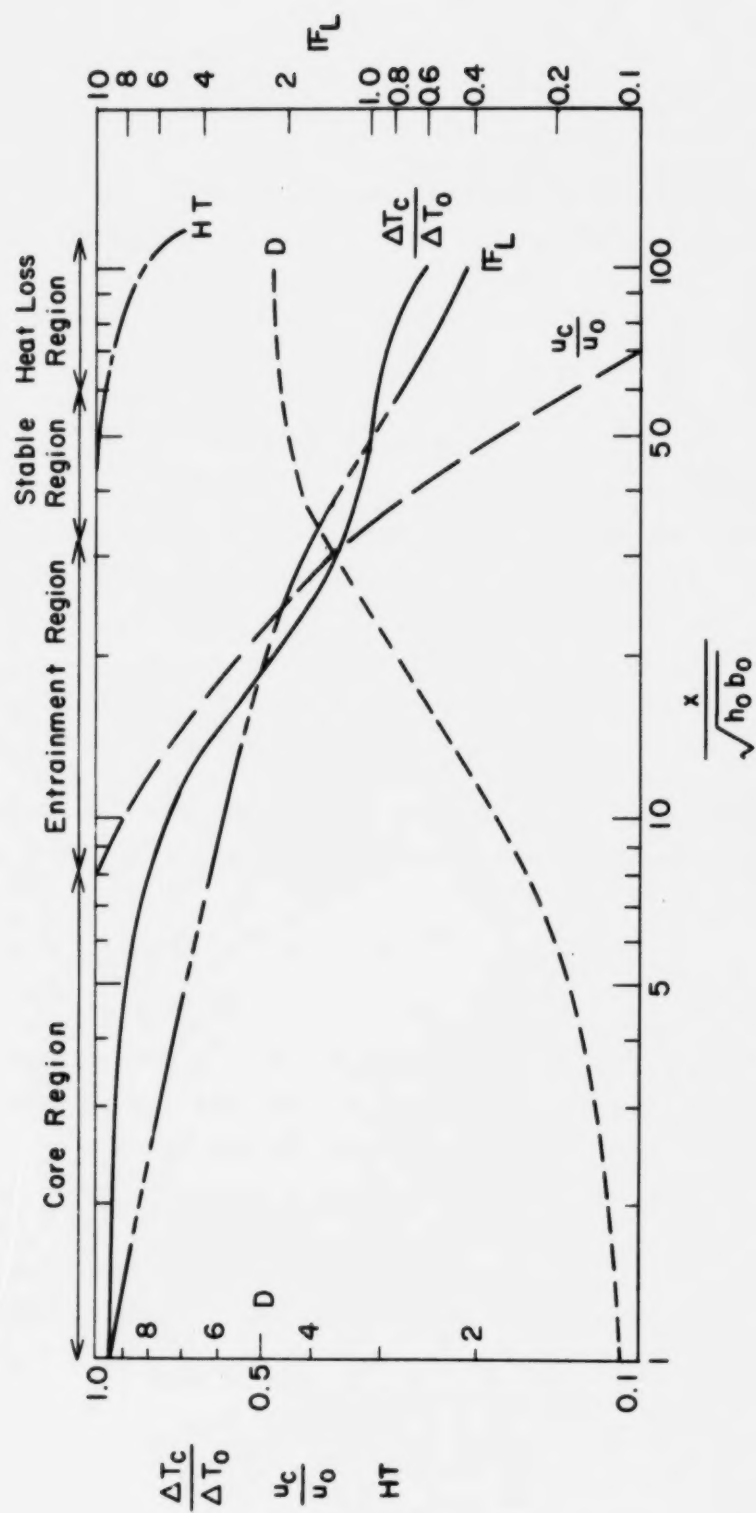


Fig. 2.3 Calculated Surface Discharge Parameters: $IF_0 = 4.4$, $A = 0.35$, $k/u_0 = 4.2 \times 10^{-5}$, $V/u_0 = 0$

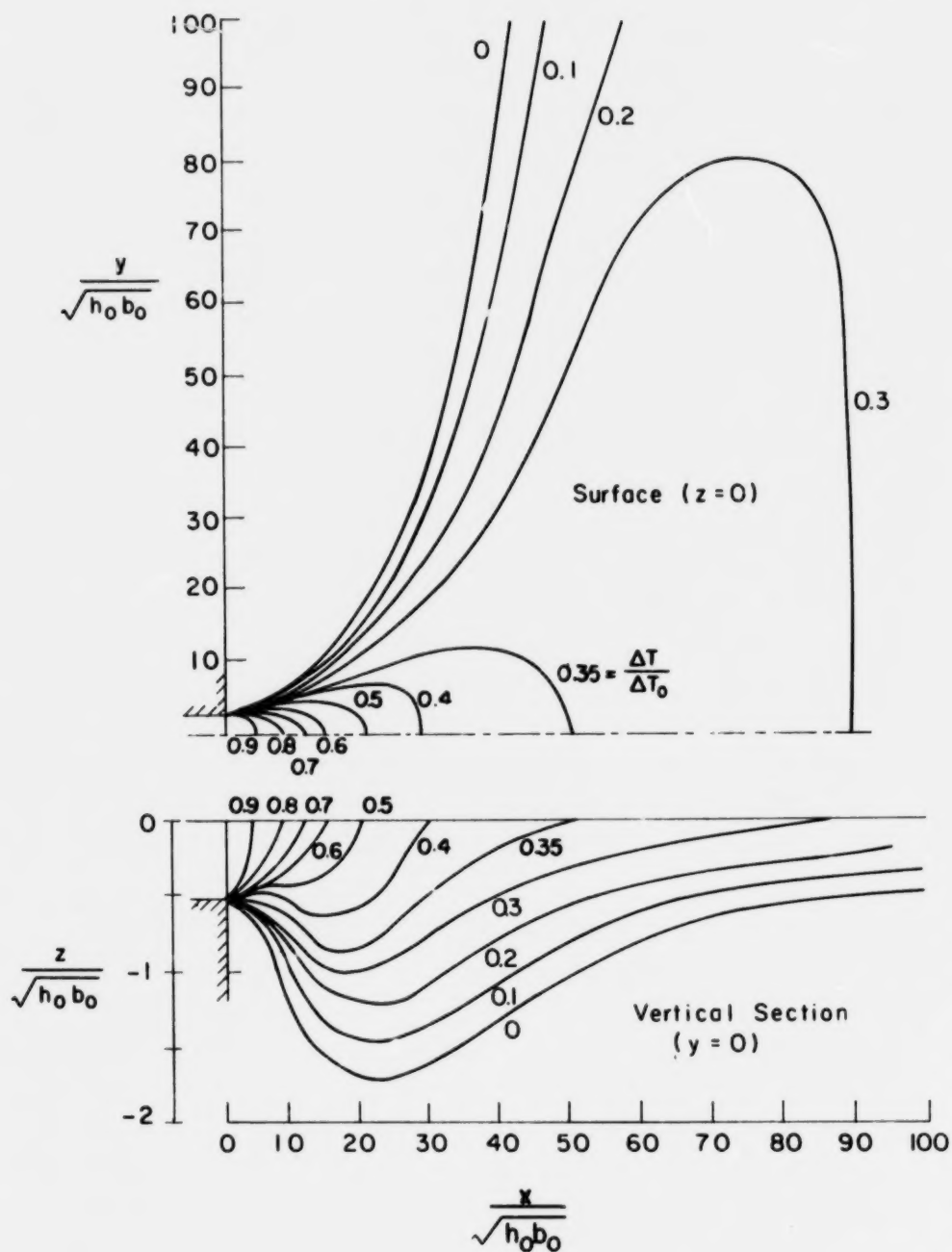


Fig. 2.4 Calculated Isotherms of $\Delta T / \Delta T_0$: $\text{Pr}_0 = 4.4$, $A = 0.35$, $k / u_0 = 4.2 \times 10^{-5}$, $V / u_0 = 0$

- 2) An entrainment region in which the centerline velocity and temperature drop sharply, approximately as $1/x$, as in a non-buoyant jet. The jet spreads vertically by turbulent processes. The lateral growth is dominated by gravitational spreading at a much greater rate than the vertical turbulent spread. Because of this large ratio of lateral to vertical spread, the jet reaches a maximum depth beyond which the bottom boundary rises to maintain mass conservation (see Figure 2.4). Local densimetric Froude numbers in this region decrease rapidly and the dilution rises sharply as a result of entrainment (see Figure 2.3). Surface heat loss remains negligible in this region, i.e., $HT = 1.0$.
- 3) A stable region in which vertical entrainment is inhibited by vertical stability as indicated by the local densimetric Froude number which is of order one or less. The jet depth continues to decrease because of lateral spreading. The small jet depths reduce the lateral entrainment, and the dilution and centerline temperature remain relatively constant in this region. The centerline velocity, however, drops sharply as a consequence of the large lateral spread. The surface temperature pattern is dominated by the wide, constant temperature stable region.
- 4) A heat loss region marking the end of the stable region. The lateral spread is sufficiently large to allow significant surface heat transfer and the temperature begins to fall again. Once surface heat loss becomes significant, the rate of temperature decrease is very rapid. However, at this point the centerline velocity is so low that the discharge may no longer be considered as a jet. In general it may be stated that surface heat loss, as determined by the value of k/u_o , is not an important factor in reducing the discharge temperature within the region of jet-like entrainment. Beyond the stable region, temperature distribution is controlled by passive diffusion processes acting upon the buoyant plume.

- 5) The primary effect of a current in the receiving water is to deflect the discharge without significantly affecting the dilution processes up to the point where the jet centerline velocity excess, u_c , is reduced to the magnitude of the cross current component in the centerline direction. For $u_c \leq V \cos \theta$ the ambient water may have a turbulence level comparable to the discharge turbulence and the basic assumptions of the theory will not be valid. This region is properly considered part of the far field.

The dilution and corresponding centerline temperature achieved in the stable region are of interest since the stable region constitutes a significant portion of the surface temperature distribution. Figure 2.5 is a plot of the dilution and surface temperature in the stable region as a function of IF_0 with values of A indicated. It is clear that the ultimate stable dilution in a buoyant jet depends primarily upon IF_0 and to a lesser extent upon A.

The maximum depth of the discharge, $h_{\max} / \sqrt{h_0 b_0}$, is also of interest and may be determined from the theoretical calculations. Figure 2.6 indicates that the maximum vertical penetration of the jet is a function of IF_0 with a very small dependence upon A.

For quick calculation involving the maximum jet penetration, centerline dilution, or centerline temperature rise, the results from Figure 2.5 and 2.6 can be condensed into three simplified formulas involving a new parameter, IF'_0 , defined by

$$IF'_0 = IF_0 A^{1/4} = \frac{u_0}{\sqrt{g \left(\frac{\Delta \rho_0}{\rho_a} \right) (h_0 b_0)^{1/2}}} \quad (2.23)$$

IF'_0 is merely a "Froude number" whose characteristic length is the scaling length, $(h_0 b_0)^{1/2}$, rather than a depth. The numerical results of Figures 2.5 and 2.6 are then approximated by

$$\left(\frac{\Delta T_0}{\Delta T_{cs}} \right) = \sqrt{(IF'_0)^2 + 1} \approx IF'_0 \quad (\text{for } IF'_0 > 3) \quad (2.24)$$

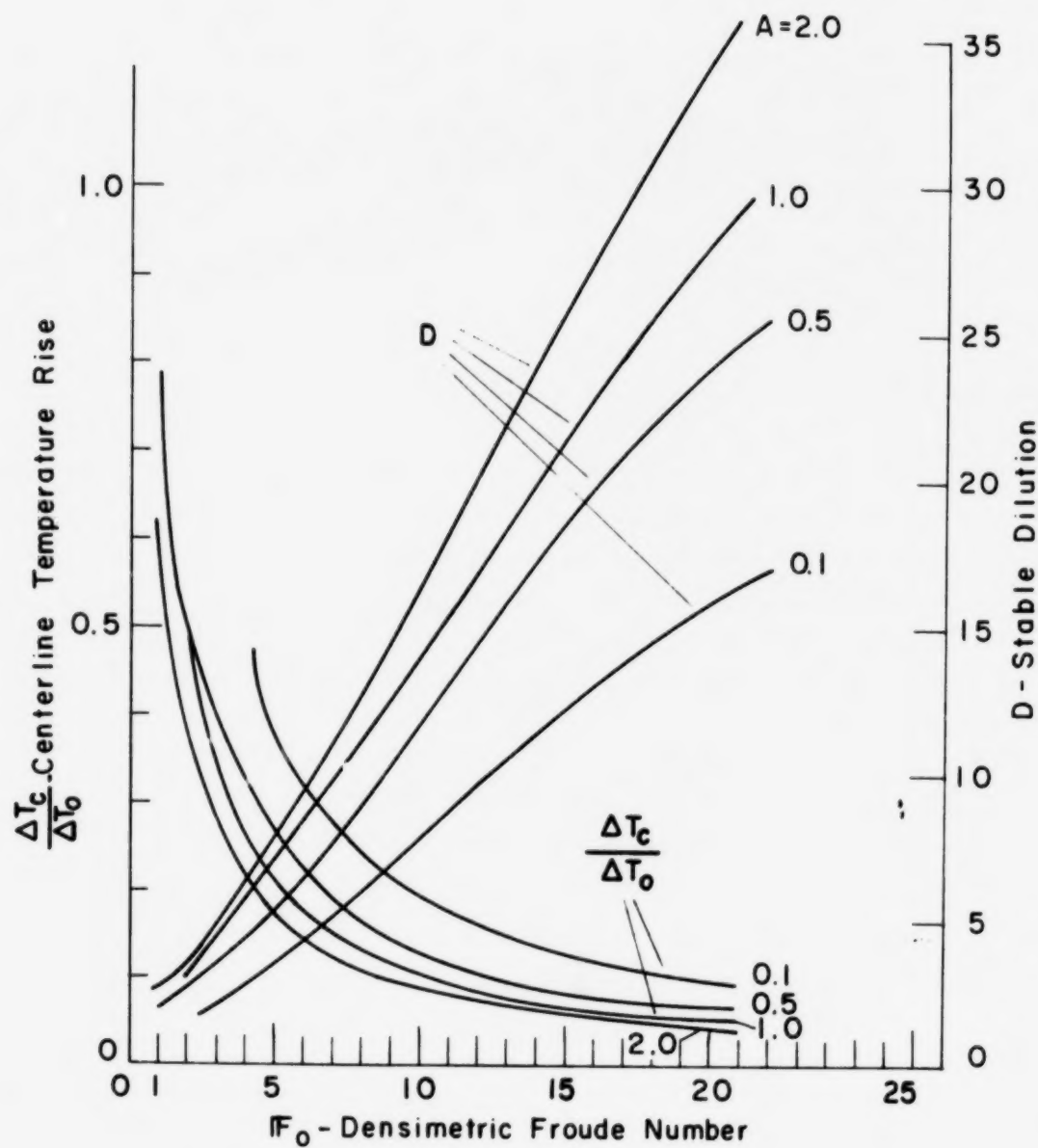


Fig. 2.5 Centerline Temperature Rise $\Delta T_c / \Delta T_0$ and Dilution, D , in the Stable Region for $k/u_0 = V/u_0 = 0$.

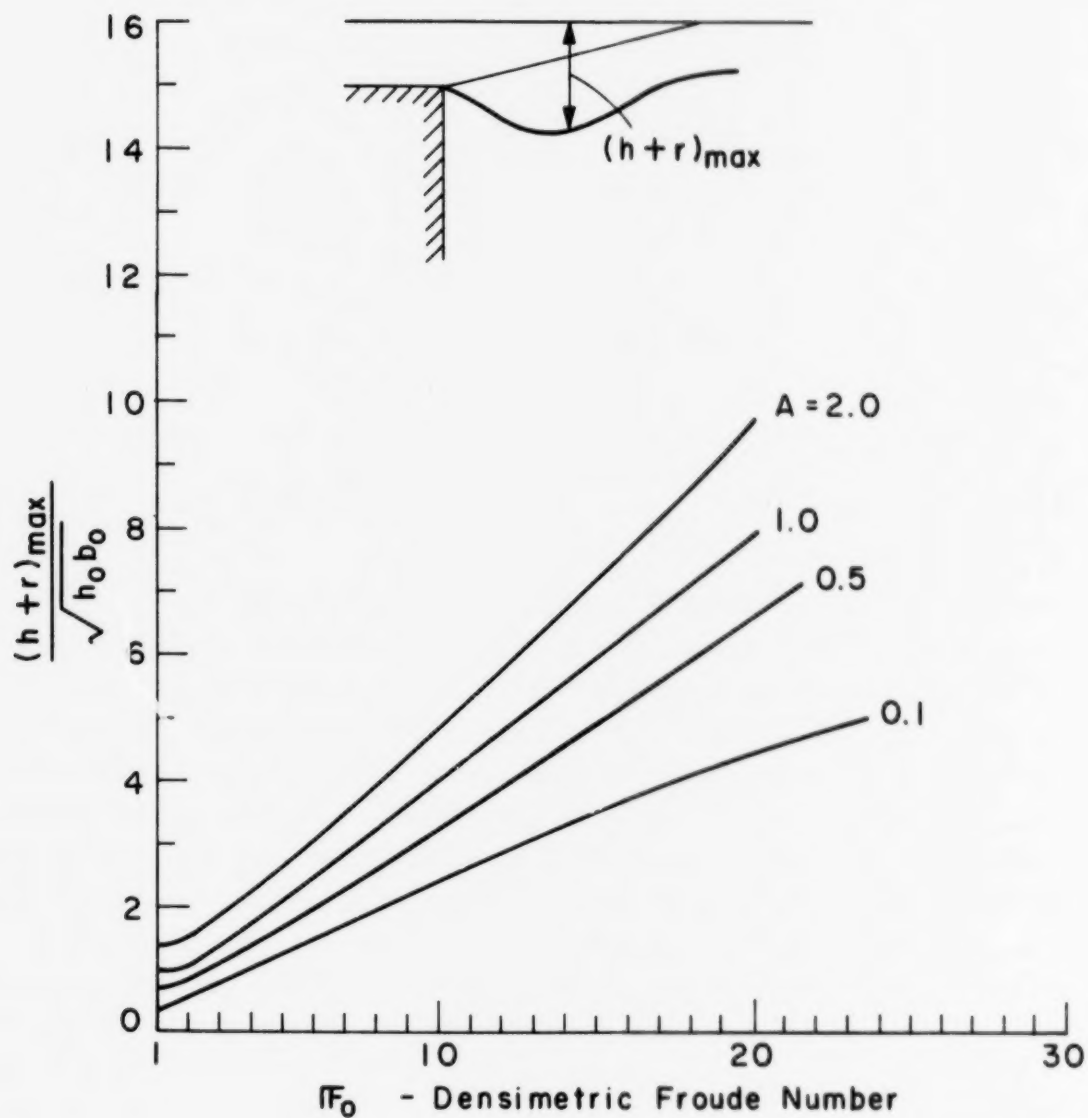


Fig. 2.6 Maximum Vertical Depth of the Jet, $\frac{(h+r)_{\max}}{\sqrt{h_0 b_0}}$
for $k/u_0 = V/u_0 = 0$

$$D_{\text{stable}} = 1.4 \sqrt{(F'_0)^2 + 1} \approx 1.4 F'_0 \text{ (for } F'_0 > 3) \quad (2.25)$$

$$\frac{(h + r)_{\text{max}}}{(h_0 b_0)^{1/2}} = 0.42 F'_0 \quad (2.26)$$

For $F'_0 > 3$, $h_{\text{max}} = (h + r)_{\text{max}}$ and (2.26) reduces to

$$\frac{h_{\text{max}}}{(h_0 b_0)^{1/2}} = 0.42 F'_0 \quad (2.27)$$

Equations 2.24 and 2.25 relate to properties of the jet in the stable region (stable centerline temperature rise and stable dilution respectively) and (2.26) relates to a maximum property of the jet. For the spatial history of the discharge or for information relating to lateral spreading or centerline deflection under a current, use of the generalized plots of the theoretical solution is suggested. Figures 2.7a to 2.7i give the basic calculated discharge properties for a range of values of F'_0 , A , and V/u_0 with k/u_0 set equal to zero. As discussed previously, a non-zero value of k/u_0 will have only secondary effect upon the temperature distribution and setting $k/u_0 = 0$ will always yield a slightly conservative result (i.e. higher temperatures). The next two chapters discuss the computer program and its application to actual discharge design.

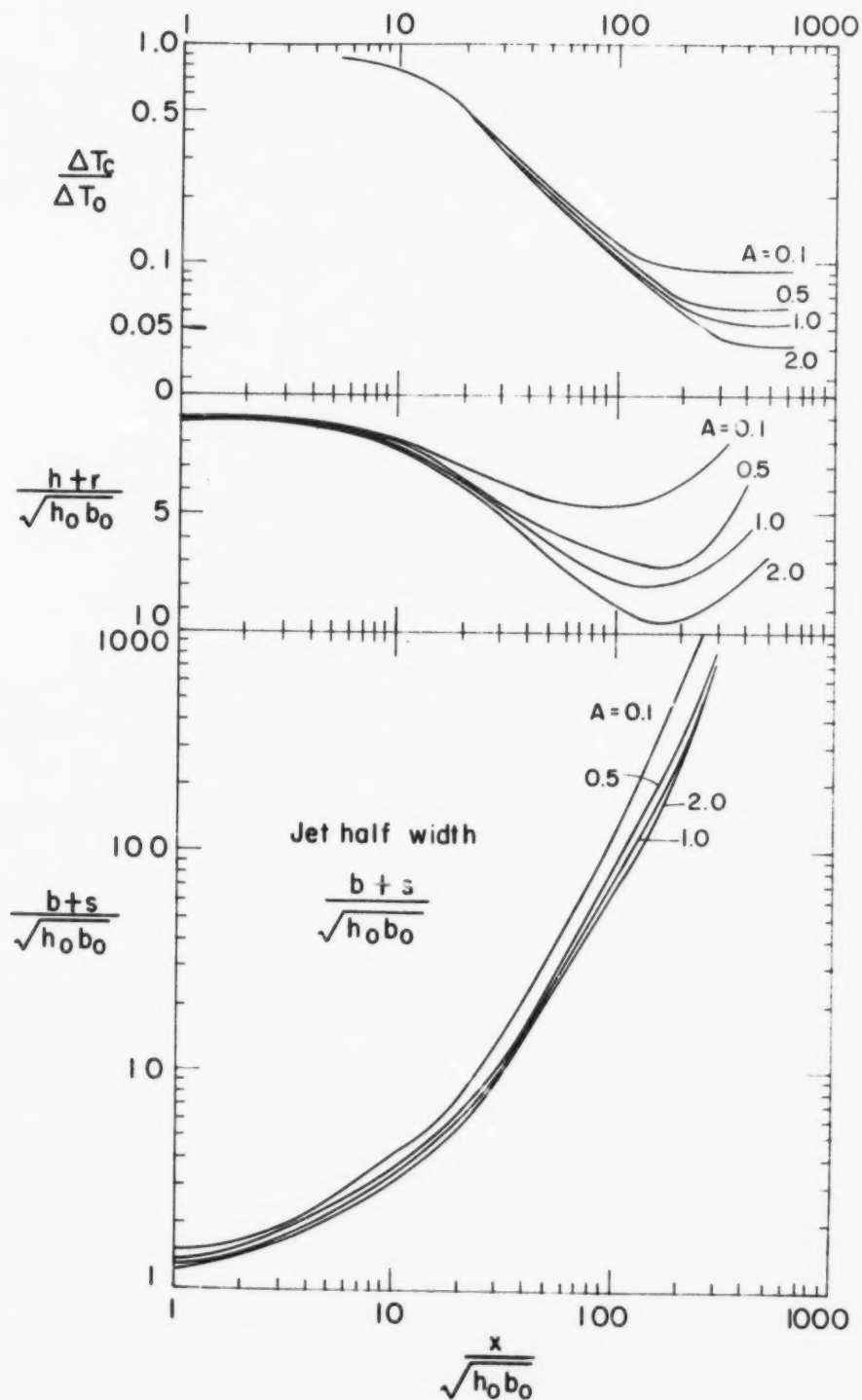


Fig. 2.7a Jet Parameters for $Pr_0 = 20.0$, $V/u_0 = 0$, $k/u_0 = 0$

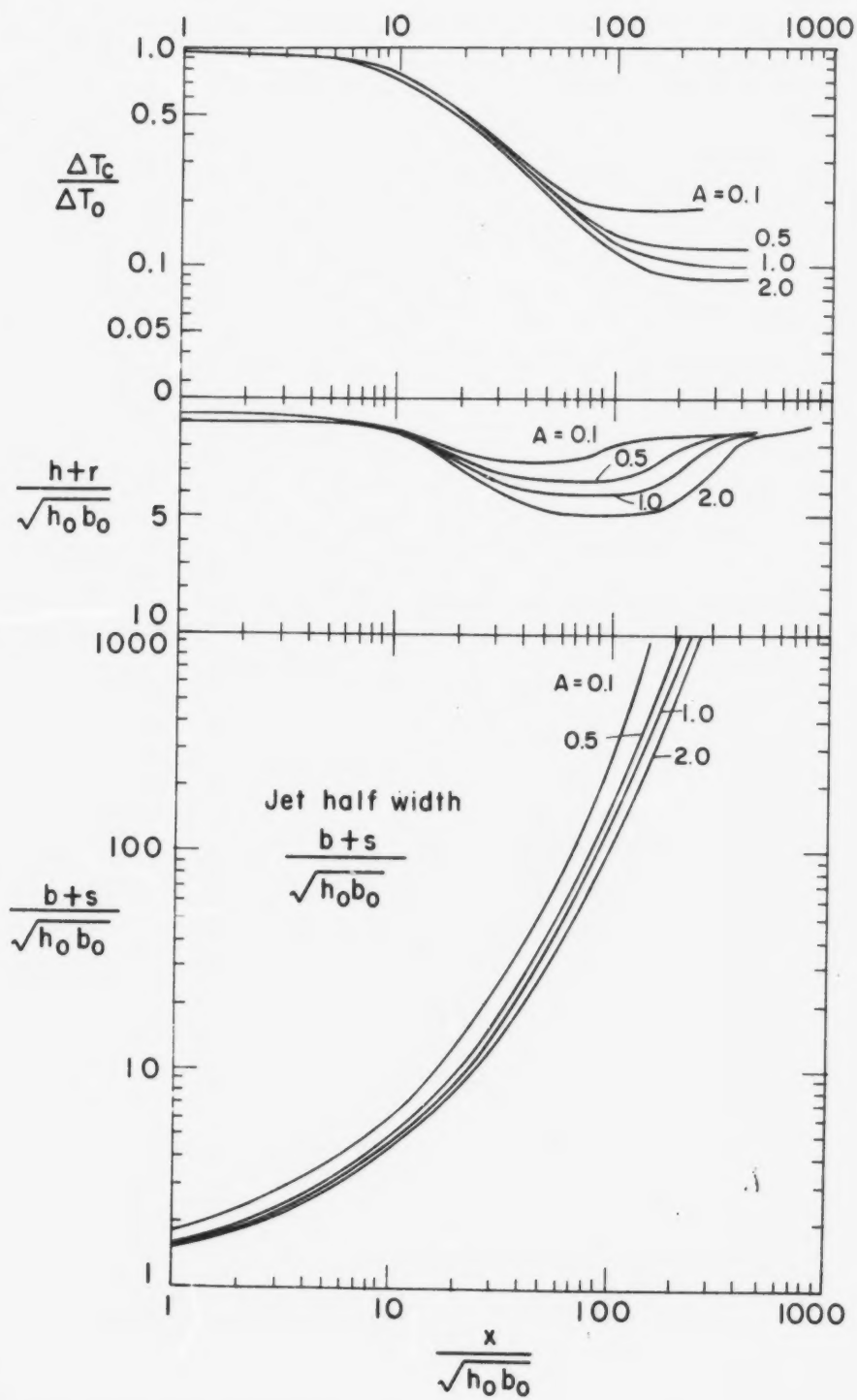


Fig. 2.7b Jet Parameters for $Pr_0 = 10.0$, $V/u_0 = 0$, $k/u_0 = 0$

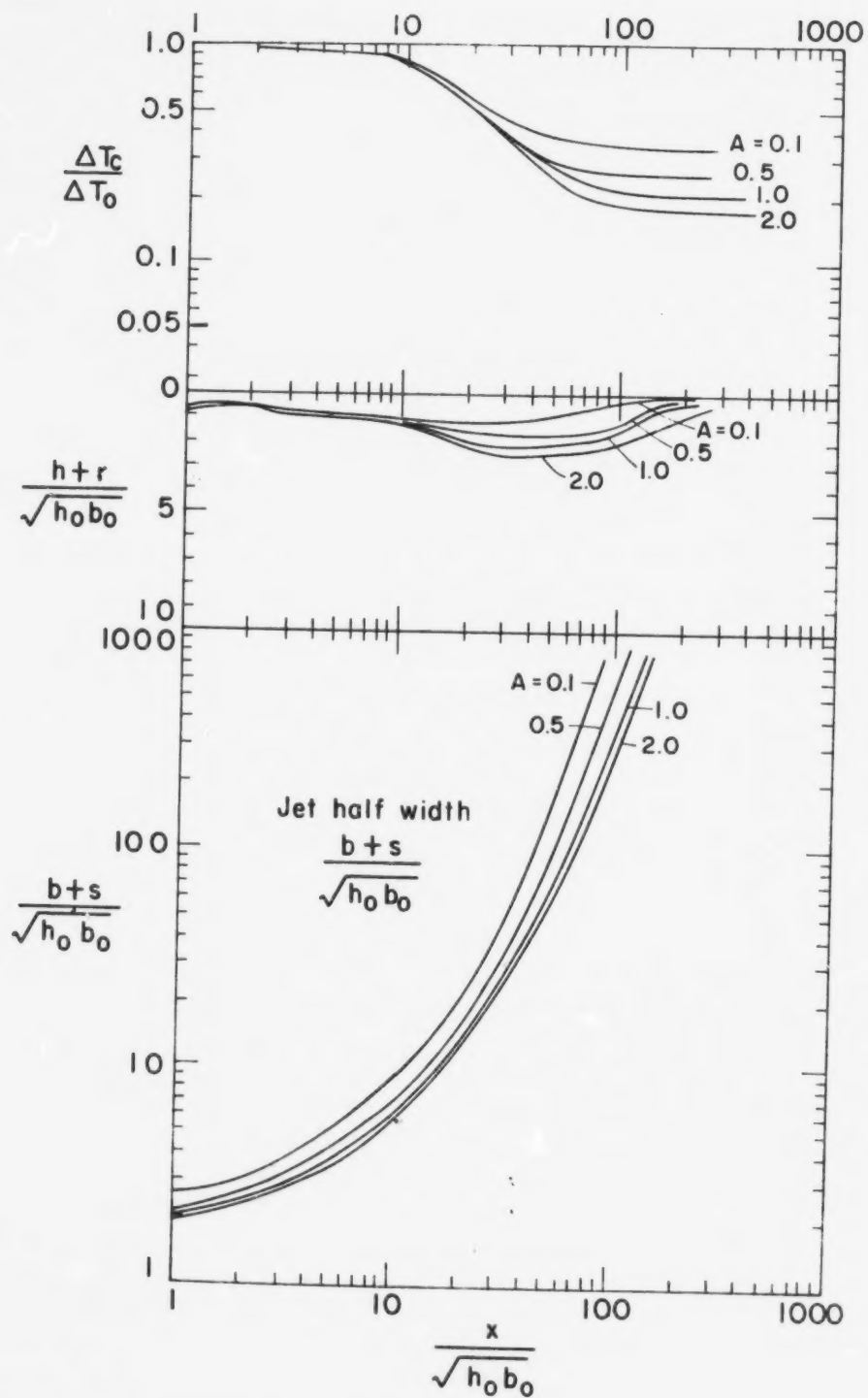


Fig. 2.7c Jet Parameters for $F_0 = 5.0$, $V/u_0 = 0$, $k/u_0 = 0$

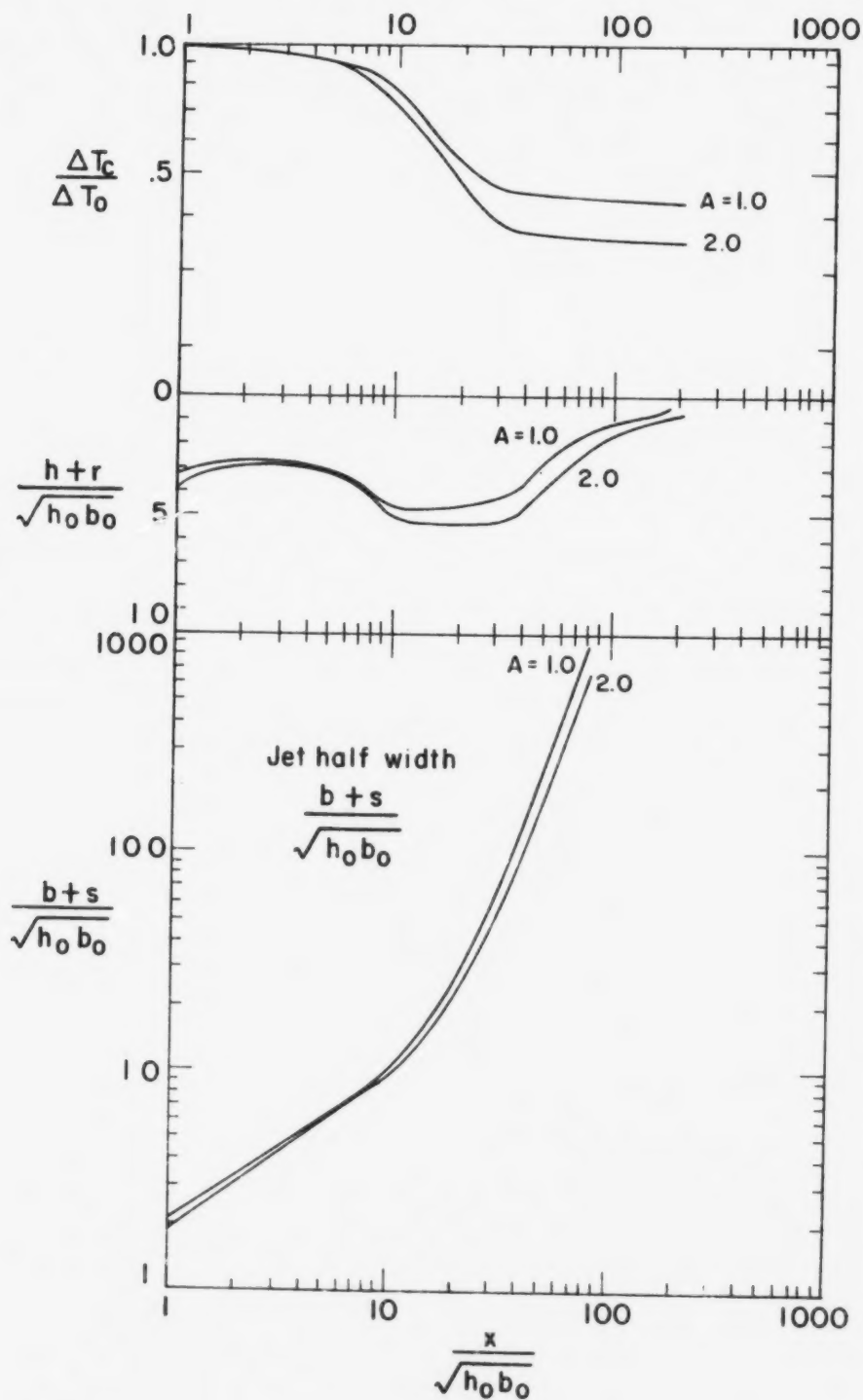


Fig. 2.7d Jet Parameters for $Fr_0 = 2.0$, $V/u_0 = 0$, $k/u_0 = 0$

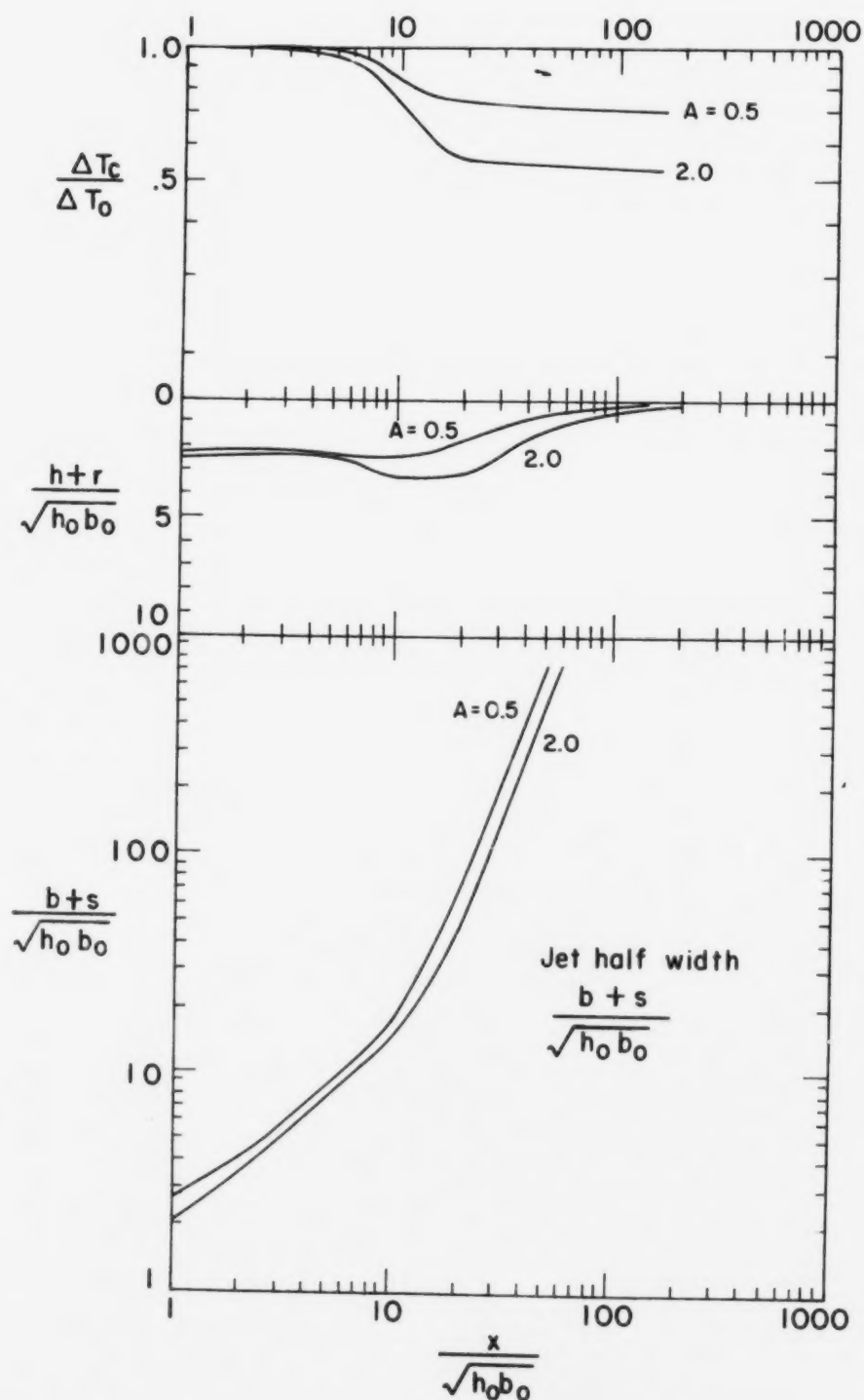


Fig. 2.7e Jet Parameters for $Fr_0 = 1.0$, $V/u_0 = 0$, $k/u_0 = 0$

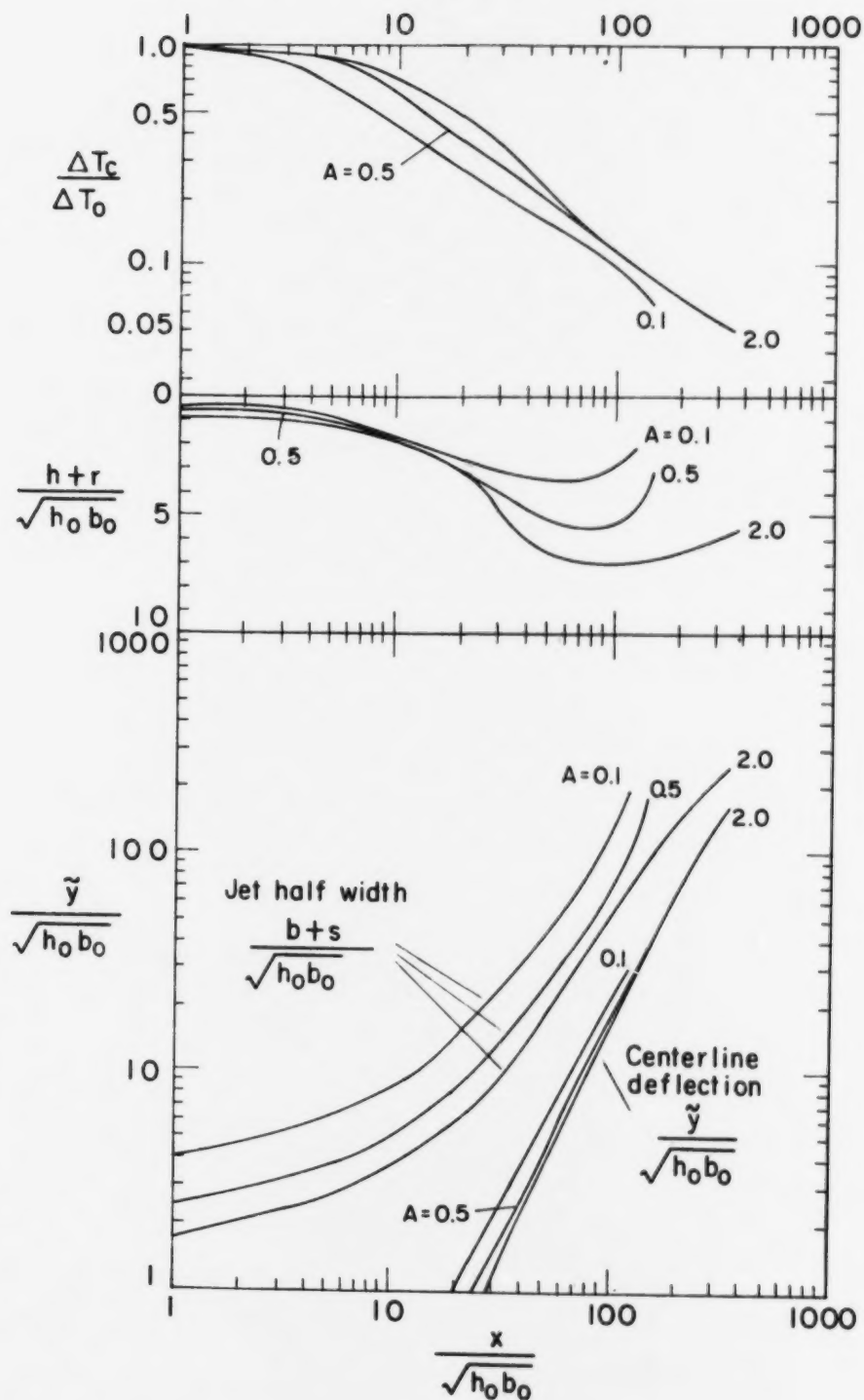


Fig. 2.7 f Jet Parameters for $IF_0 = 20.0$, $V/u_0 = 0.025$, $k/u_0 = 0$

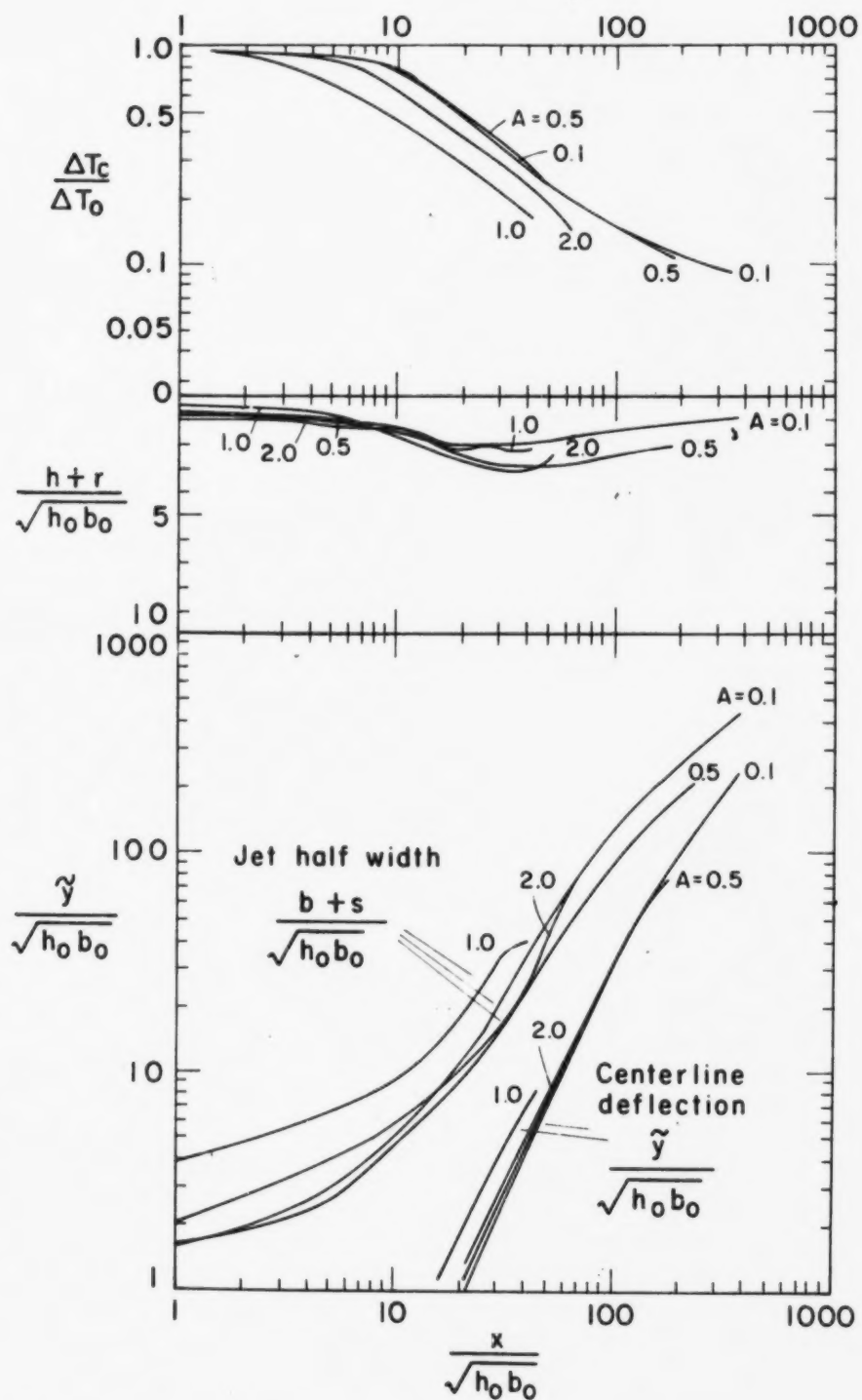


Fig. 2.7g Jet Parameters for $If_0=10.0$, $V/u_0=0.05$, $k/u_0=0$

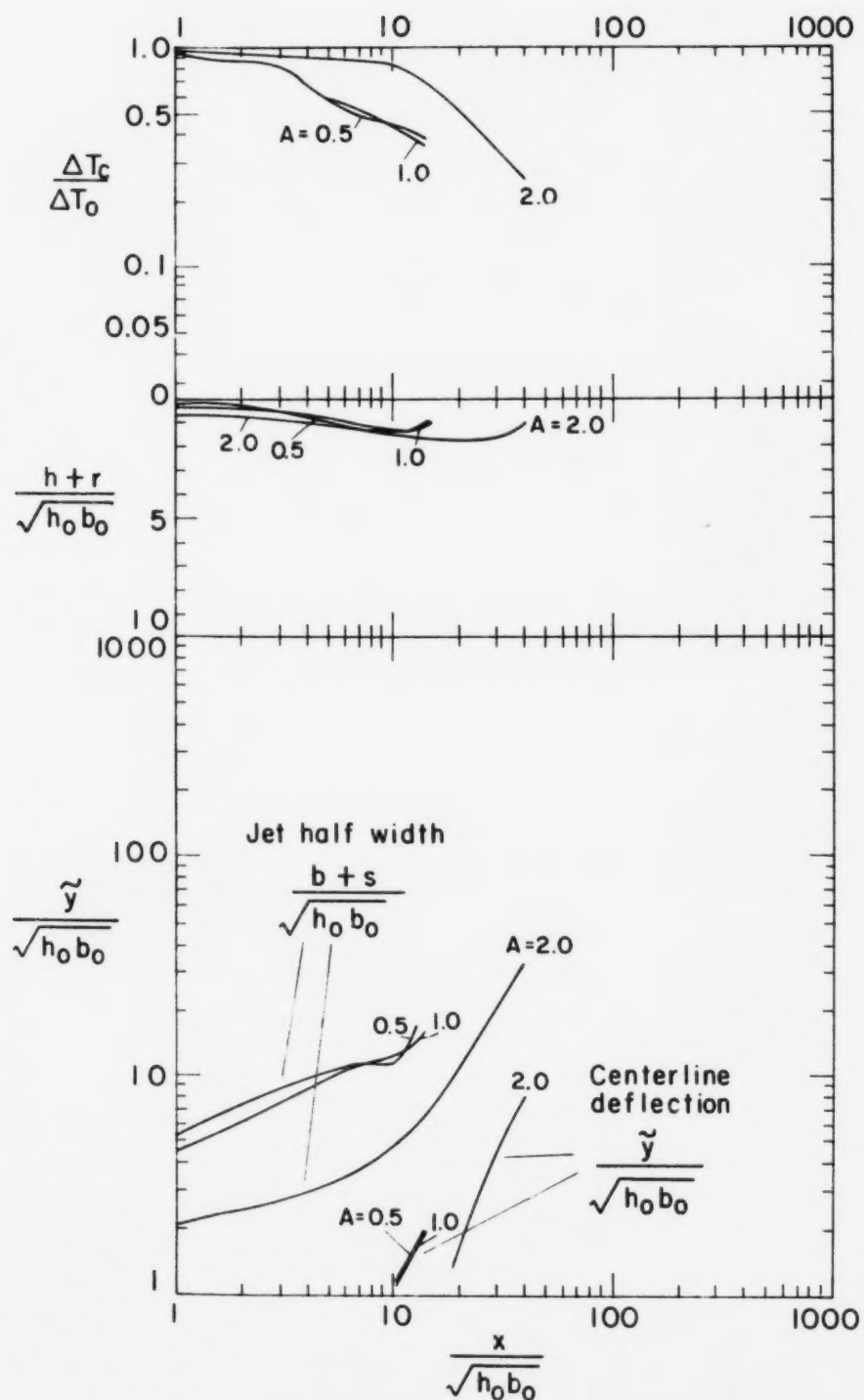


Fig. 2.7h Jet Parameters for $Pr_0 = 5.0$, $V/u_0 = 0.1$, $k/u_0 = 0$

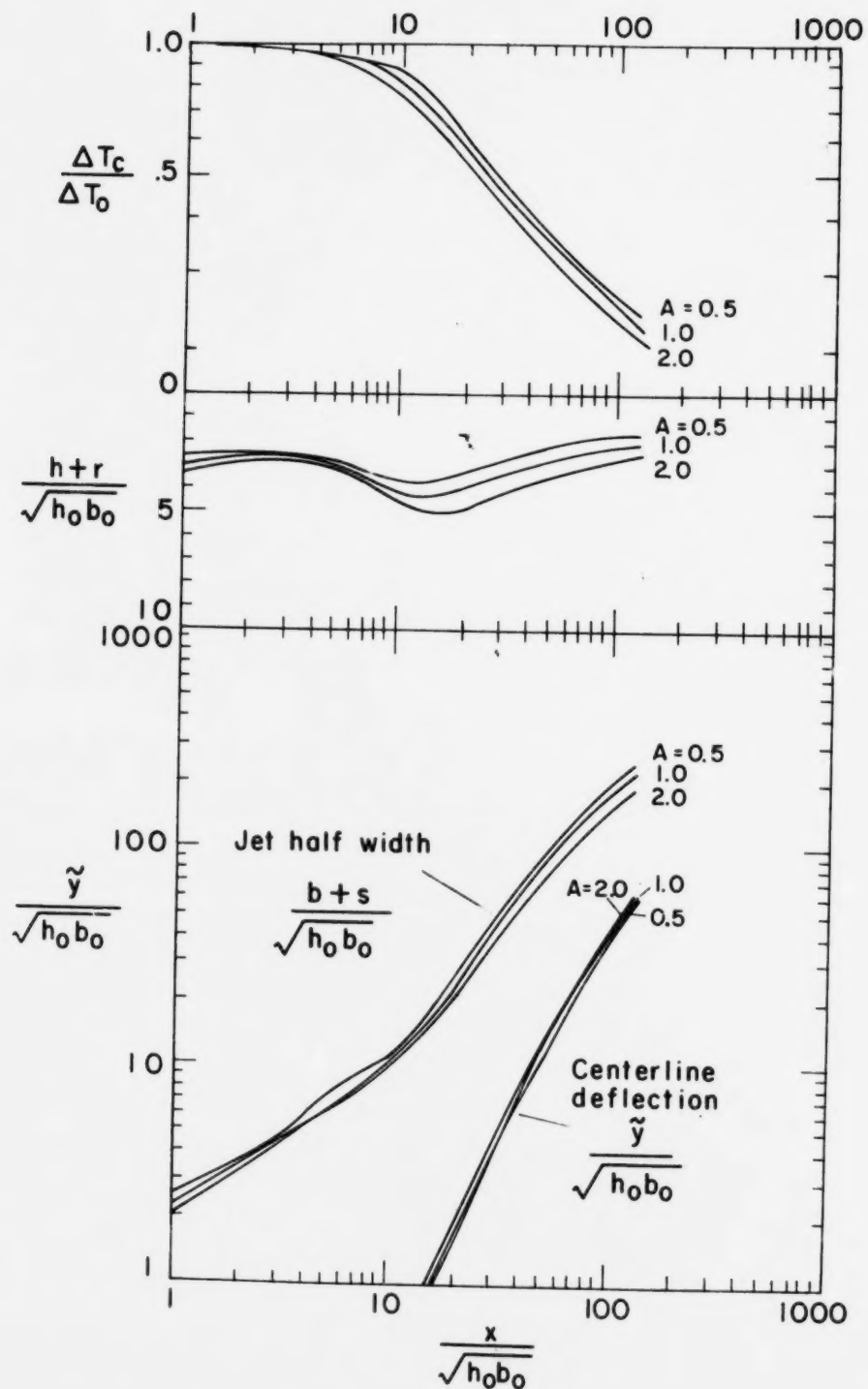


Fig. 2.7i Jet Parameters for $Fr_0 = 2.0$, $V/u_0 = 0.1$, $k/u_0 = 0$

III. The Program

This chapter is a detailed description of the program for computing the solution to the heated surface discharge equations. The solution method is described including the basic equations, and the computational scheme. Lastly, the computer program input and output format are discussed.

3.1 Dimensionless Equations

The computations are performed with the following dimensionless variables:

$$\begin{aligned}
 \bar{x} &= x / \sqrt{h_o b_o} \\
 \bar{\tilde{x}} &= \tilde{x} / \sqrt{h_o b_o} \\
 \bar{\tilde{y}} &= \tilde{y} / \sqrt{h_o b_o} \\
 \bar{u} &= u_c / u_o \\
 \bar{\Delta T} &= \Delta T_c / \Delta T_o \\
 \bar{h} &= h / \sqrt{h_o b_o} \\
 \bar{b} &= b / \sqrt{h_o b_o} \\
 \bar{r} &= r / \sqrt{h_o b_o} \\
 \bar{s} &= s / \sqrt{h_o b_o} \\
 \bar{V} &= V / u_o \\
 \theta &
 \end{aligned}
 \tag{3.1}$$

The equation set in Table 2.1 is reduced by eliminating the internal velocities v_s , u_b , w_r and w_h as follows:

- a) Sum all the mass conservation equations to form a
total mass conservative equation:

$$\frac{d}{d\bar{x}} \left[\bar{u}(\bar{s} + \bar{b}I_1)(\bar{r} + \bar{h}I_1) + \bar{V} \cos \theta (\bar{s} + \bar{b})(\bar{r} + \bar{h}) \right] - \bar{u} \left[\alpha_{sz}(\bar{s} + \bar{b}I_1) - \alpha_y(\bar{r} + \bar{h}I_1) \right] = 0 \tag{3.2}$$

b) Sum all the momentum conservation equations to form a
total momentum conservation equation:

$$\begin{aligned} \frac{d}{d\bar{x}} \left[\bar{u}^2 (\bar{s} + \bar{b}I_2) (\bar{r} + \bar{h}I_2) + 2\bar{u}\bar{v}\cos\theta (\bar{s} + \bar{b}I_1) (\bar{r} + \bar{h}I_1) + \bar{v}^2 \cos^2\theta (\bar{s} + \bar{b}) (\bar{r} + \bar{h}) \right. \\ \left. + F_0^{-2} A^{-1/2} \bar{\Delta T} (\bar{s} + \bar{b}I_3) \left(\frac{1}{2} \bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2I_4 \right) \right] - \bar{u}\bar{v}\cos\theta \left[\alpha_{sz} (\bar{s} + \bar{b}I_1) - \alpha_y (\bar{r} + \bar{h}I_1) \right] = 0 \end{aligned} \quad (3.3)$$

c) The total heat equation:

$$\begin{aligned} \frac{d}{d\bar{x}} \left[\bar{u} \bar{\Delta T} (\bar{s} + \bar{b}I_7) (\bar{r} + \bar{h}I_7) + \bar{\Delta T} \bar{v}\cos\theta (\bar{s} + \bar{b}I_3) (\bar{r} + \bar{h}I_3) \right] \\ + \left[\frac{k}{u_0} \right] \bar{\Delta T} (\bar{s} + \bar{b}I_3) = 0 \end{aligned} \quad (3.4)$$

d) The y-momentum spreading equation:

$$\begin{aligned} \frac{d}{d\bar{x}} \left[\left[\frac{d\bar{b}}{d\bar{x}} - \epsilon \right] \left[\bar{u}^2 \bar{b}I_6 (\bar{r} + \bar{h}I_2) + 2\bar{u}\bar{v}\cos\theta \bar{b}I_5 (\bar{r} + \bar{h}I_1) + \bar{v}^2 \cos^2\theta \bar{b} (\bar{r} + \bar{h}) \right] \right] \\ - F_0^{-2} A^{-1/2} \bar{\Delta T} \left(\frac{1}{2} \bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2I_4 \right) = 0 \end{aligned} \quad (3.5)$$

e) The y-momentum deflection equation:

$$\frac{d\theta}{d\bar{x}} + \frac{\bar{u}\bar{v}\sin\theta \left[\alpha_{sz} (\bar{s} + \bar{b}I_1) + \alpha_y (\bar{r} + \bar{h}I_1) \right]}{\bar{u}^2 (\bar{s} + \bar{b}I_2) (\bar{r} + \bar{h}I_2) + 2\bar{u}\bar{v}\cos\theta (\bar{s} + \bar{b}I_1) (\bar{r} + \bar{h}I_1) + \bar{v}^2 \cos^2\theta (\bar{s} + \bar{b}) (\bar{r} + \bar{h})} = 0 \quad (3.6)$$

f) Combine the region 1 mass and momentum conservation equations to eliminate v_s and w_r :

$$(\bar{u} + \bar{v} \cos \theta) \frac{d}{d\bar{x}} (\bar{u} + \bar{v} \cos \theta) + F_o^{-2} A^{-1/2} \left[\frac{1}{2} \bar{r} \frac{d\bar{\Delta T}}{d\bar{x}} + \bar{\Delta T} \frac{d\bar{r}}{d\bar{x}} + I_3 \frac{d\bar{\Delta T}}{d\bar{x}} \bar{h} \right] = 0$$

(3.7)

g) Combine the momentum and mass conservation equations from regions 1, 2, and 3 to eliminate v_s , v_b , w_t and w_h :

$$\begin{aligned} & (\bar{r}\bar{b} + \bar{s}\bar{h}) \left[(\bar{u}\bar{I}_2 + \bar{v} \cos \theta I_1) \frac{d\bar{u}}{d\bar{x}} + (\bar{u}(2I_1 - \frac{I_2}{I_1}) + \bar{v} \cos \theta) \frac{d\bar{v} \cos \theta}{d\bar{x}} \right] \\ & + \bar{u} \bar{v} \cos \theta (I_1 - \frac{I_2}{I_1}) (\bar{s} \frac{d\bar{h}}{d\bar{x}} + \bar{r} \frac{d\bar{b}}{d\bar{x}}) + \bar{u} (1 - \frac{I_2}{I_1}) \frac{d}{d\bar{x}} [\bar{r}\bar{s} (\bar{u} + \bar{v} \cos \theta)] \\ & + F_o^{-2} A^{-1/2} \left[\bar{s} (I_4 \frac{d\bar{\Delta T}}{d\bar{x}} \bar{h}^2 + I_3 \bar{\Delta T} \bar{h} \frac{d\bar{r}}{d\bar{x}}) + \frac{1}{2} I_3 \frac{d\bar{\Delta T}}{d\bar{x}} \bar{b} \bar{r}^2 + I_3 \bar{r}^2 \frac{d\bar{\Delta T}}{d\bar{x}} \bar{b} \bar{h} + \right. \\ & \left. \bar{\Delta T} (\frac{1}{2} \bar{r}^2 + \bar{r}\bar{h} I_3) \frac{d\bar{s}}{d\bar{x}} \right] + \bar{u} (\alpha_{sz} \bar{s} - \alpha_y \bar{r}) \frac{I_2}{I_1} = 0 \end{aligned} \quad (3.8)$$

h) The geometrical relationships:

$$\frac{d\bar{x}}{d\bar{x}} - \sin \theta = 0$$

(3.9)

$$\frac{d\bar{y}}{d\bar{x}} - \cos \theta = 0$$

Using the similarity functions defined in the theory, the integration constants have the values:

$$\begin{aligned}
 I_1 &= .4500 \\
 I_2 &= .3160 \\
 I_3 &= .6000 \\
 I_4 &= .2143 \\
 I_5 &= .2222 \\
 I_6 &= .1333 \\
 I_7 &= .3680
 \end{aligned}
 \tag{3.10}$$

The entrainment coefficients are as follows assuming $\epsilon_0 = .22$:

$$\begin{aligned}
 \alpha_z &= .0295 & \bar{r} > 0 \\
 \alpha_z &= .0495 & \bar{r} = 0
 \end{aligned}
 \quad
 \alpha_{sz} = \alpha_z \exp \left[-5 \frac{\bar{\Delta T} \bar{h}}{\bar{u}^2} \bar{F}_0^{-2} A^{-1/2} \right]$$

$$\begin{aligned}
 \alpha_y &= -.0295 & \bar{s} > 0 \\
 \alpha_y &= -.0495 & \bar{s} = 0
 \end{aligned}
 \tag{3.11}$$

It should be noted that the computer program may be used for other choices of similarity functions and spreading rate than those assumed here by simply changing the program statements which specify the values of I_1 - I_7 and ϵ_0 . The entrainment coefficients are automatically computed from these variables using equations (2.18)

The above nine equations may be solved for the $\bar{x}$ derivatives of $\bar{x}$, $\bar{y}$, $\bar{u}$, $\bar{\Delta T}$, $\bar{h}$, $\bar{b}$, $\bar{r}$, $\bar{s}$, and θ . (Note that $\bar{V}$ is given as an input and thus is known. The "initial" conditions at $\bar{x} = 0$ are:

$$\begin{aligned}
 \bar{x} = \bar{y} &= 0 & \bar{h} = \bar{b} &= 0 \\
 \bar{u} &= 1 - \bar{V} \cos \theta_0 \quad (\bar{V} \text{ taken at origin}) & \bar{r} &= A^{1/2} \\
 \bar{\Delta T} &= 1 & \bar{s} &= A^{-1/2} \\
 & & \theta &= \theta_0
 \end{aligned}
 \tag{3.12}$$

3.2 The Computational Scheme

The computer program is conceptually simple. After all inputs are read the variables are initialized to their values at the discharge origin. The solution proceeds by advancing along the discharge centerline (increasing $\bar{x}$) using the calculated derivatives of each variable to calculate its behavior. The differential equation technique is a fourth order Runge-Kutta scheme which has been taken, in modified form, from the IBM Scientific Subroutine Package. The program consists of a main program and five subroutines:

MAIN: This program reads the input data, sets up the initial conditions for each calculation, and calls the subroutine SRKGS which performs the calculations.

SRKGS: This subroutine is a modified version of the IBM Scientific Subroutine Package DRKGS for solving a system of differential equations. The form of the equation system is:

$$[a_{ij}] \frac{dy_j}{dx} = c_i \quad (3.13)$$

where a_{ij} is a coefficient matrix, y_j is a vector of the variables ($\bar{u}$, ΔT , etc.) and c_i is a vector of the constants in the i equations. Subroutine SRKGS advances the solution by successively calling subroutine FCT which solves the system of equations for $\frac{y_j}{dx}$. The results of the calculations are periodically printed by calling subroutine OUTP.

FCT: This subroutine uses the current values of the variables, computes the coefficient matrix, a_{ij} , the vector c_i , and solves the resulting system of linear equations for $\frac{y_j}{dx}$ by calling routine SGELG. The following details concerning this routine may be of interest:

- a) To reduce the equation set to first order the equation $\frac{d}{d\bar{x}} \left[\frac{db}{d\bar{x}} \right] = \frac{d^2 b}{dx^2}$ is added where $\frac{d^2 b}{dx^2}$ is computed from the y-spreading Equation (3.5) and $\frac{db}{d\bar{x}}$ becomes a variable.
- b) Because of their relatively simple form, the y-deflection Equation (3.6) and the geometrical relationships (3.9) are not included in the matrix

solved by SGELG but are solved separately within FCT.

- c) When an ambient current is present, ($V \neq 0$) the value of ϵ is computed by setting $\overline{\Delta T} = 0$ and solving a reduced set of equations from which the heat equation (3.4) is omitted and the y-spreading equation (3.5) is replaced by $\frac{db}{dx} = \frac{dh}{dx}$. The full buoyant equation set is then solved using the calculated value of ϵ . When there is no ambient current, ($V=0$) the spreading rate is $\epsilon_0 = .22$, and the full equation set is solved directly.

SGELG: This subroutine is a modified form of the IBM Scientific Subroutine Package routine DGELG. It solves the system of linear equations by Gauss-Jordan reduction.

OUTP: The values of the calculated variables (y_j) are printed in formatted form. Certain other quantities of interest are also printed (see Section 3.4).

CROSS: This subroutine is called by the other routines whenever the velocity of the cross flow is required. The routine uses the input values V_1 to V_8 to compute the cross flow as a function of $\bar{x}$. This subroutine may be modified by any user to any particular functional form with the only requirement being that the routine place the value of the cross flow at the current value of $\bar{x}$ in V and the value of $\frac{d\bar{V}}{d\bar{x}}$ in DV . The cross flow function used in the present version is described in the next section.

The computations for a particular case will be terminated for one of the following reasons:

- 1) the limit on $\bar{x}$ specified by the input has been reached (see next section). This is the normal termination.
- 2) $\bar{u} \leq \bar{V} \cos \theta$: This termination occurs when the centerline velocity excess is reduced to the same magnitude as the ambient current in which case the basic assumptions of the theory are not valid.
- 3) $\bar{u} \leq .02$: The limiting value .02 for $\bar{u}$ is an arbitrarily chosen small number below which the assumptions of jet-like behavior are not valid.

- 4) The total dimensionless momentum, $M = \bar{u}^2(\bar{r} + \bar{h}I_2)(\bar{s} + \bar{b}I_2) + 2\bar{u}\bar{V}\cos\theta(\bar{r} + \bar{h}I_1)(\bar{s} + \bar{b}I_1) + \bar{V}^2\cos^2\theta(\bar{r} + \bar{h})(\bar{s} + \bar{b}) + F_o^{-2}A^{-1/2}\frac{\Delta T}{\bar{s} + \bar{b}I_3}(\frac{1}{2}\bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2I_4)$, which should be constant for $\frac{\bar{V}}{\bar{u}_o} = 0$ and nearly constant for $\frac{\bar{V}}{\bar{u}_o}$ near zero has deviated from its initial value by more than 25%. This termination indicates that the computation is accumulating a large numerical error.

3.3 Input Formats

	Input Data	Format
	Number of cases to be calculated	I3
One Set for each Calculation	$F_o, A, k/u_o, \theta_o, \bar{x}_L, ERR, STEP$	2F10.5, F10.7, 4F10.5
	$V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8$	8F10.5

where $F_o = \text{initial densimetric Froude number} = \frac{u_o}{\sqrt{\frac{\Delta\rho_o}{\rho_a}gh_o}} = \frac{u_o}{\sqrt{\beta\Delta T_o gh_o}}$

$A = \text{aspect ratio} = h_o/b_o$

$k/u_o = \text{surface heat loss parameter}$

$\theta_o = \text{initial angle (in degrees) between the discharge centerline and the boundary of the ambient region}$

$\bar{x}_L = \text{the value of } x/\sqrt{h_o b_o} \text{ at which the program should terminate for this case}$

$ERR = \text{maximum allowable average error in the variables at each time step}$

$STEP = \text{interval of } \bar{x} \text{ at which variable values should be printed}$

$V_1 - V_8 = \text{constants describing the cross flow. In the present program version only } V_1 - V_5 \text{ are used as follows}$

$$\bar{V} = V_1 + V_2 \exp \left[-V_3 \left[V_4 \bar{x} - V_5 \right]^2 \right]$$

$$\frac{d\bar{V}}{d\bar{x}} = 2(V_4 \bar{x} - V_5)V_2V_3V_4 \exp \left[-V_3(V_4 \bar{x} - V_5)^2 \right]$$

A sample input listing is given in Appendix III.

3.4 Output Format

The output is paged for each calculation with the basic input variables printed on the heading of each page. The values of the variables are printed in column form under the following headings

$$\begin{aligned}
 X &= \bar{x} \\
 H &= \bar{h} \\
 B &= \bar{b} \\
 R &= \bar{r} \\
 S &= \bar{s} \\
 IF_L &= IF_O A^{1/4} / \left(\frac{\Delta T \bar{h}}{\bar{u}^2} \right)^{1/2} \\
 Q &= \bar{u}(\bar{r} + \bar{h}I_1)(\bar{s} + \bar{b}I_1) + \bar{V}\cos\theta (\bar{r} + \bar{h})(\bar{s} + \bar{b}) \\
 M &= \bar{u}^2(\bar{r} + \bar{h}I_2)(\bar{s} + \bar{b}I_2) + 2\bar{u}\bar{V}\cos\theta(\bar{r} + \bar{h}I_1)(\bar{s} + \bar{b}I_1) \\
 &\quad + \bar{V}^2\cos^2\theta(\bar{r} + \bar{h})(\bar{s} + \bar{b}) + IF_O^{-2} A^{-1/2} \Delta T(\bar{s} + \bar{b}I_3) \left(\frac{1}{2}\bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2I_4 \right) \\
 U &= \bar{u} \\
 T &= \Delta T \\
 HT &= \Delta T [\bar{u}(\bar{r} + \bar{h}I_7)(\bar{s} + \bar{b}I_7) + \bar{V}\cos\theta(\bar{r} + \bar{h})(\bar{s} + \bar{b})] \\
 V &= \bar{V} \\
 XP &= \bar{x} \\
 YP &= \bar{y} \\
 THD &= \theta \\
 TM &= \int_0^{\bar{x}} \frac{d\bar{x}}{\bar{u}}
 \end{aligned}$$

An example of program output is shown in Appendix III.

IV. Applications

The theoretical model presented in the preceding sections permits determination of the behavior of a heated surface discharge as a function of relatively few controlling parameters: IF_o , h_o/b_o , k/u_o , V/u_o . The geometry of actual field configurations are rarely as simple as those assumed in the theory and judgement must be applied in the schematization of the discharge to a form for which the dimensionless parameters may be given. Similarly, the output of the program must be interpreted in the light of the basic assumptions of the model. The following sections discuss in detail the generation of input data for the computations and the construction of the temperature distribution from the program output. Finally, a case study is presented as an example of the use of the theory and computer program.

4.1 Schematization

This section is a discussion of the data requirements and schematization techniques for preparing input to the program. The following are the physical data needed as input to the theoretical calculations.

The ambient temperature, T_a , is assumed in the theory to be constant in space and time. Actual ambient receiving water temperatures are often stratified vertically or horizontally and may be unsteady due to wind, tidal action or diurnal variations in solar heating. The natural stratification of the ambient water may be increased by accumulation of heat at the water surface if the discharge is in a semi-enclosed region. The value of the ambient temperature, for a given initial temperature rise, ΔT_o , determines the initial density difference between the discharged and ambient water. Also the effectiveness of the entrainment of ambient water in reducing the discharge temperatures is a function of the ambient stratification.

If the temperature differences resulting from ambient stratification in the vicinity of the discharge are the same magnitude as the initial discharge temperature rise, the theoretical model of this study should not be applied without further development to account for the ambient stratification. The theory is valid if an ambient temperature, T_a , may be chosen which is representative of the receiving water temperatures, that is, if the temporal or spatial variations in ambient temperature do not differ from T_a by more than a few degrees Fahrenheit.

The initial discharge temperature rise, ΔT_o , is determined from the discharge temperature, T_o , and the chosen ambient temperature, T_a . The value of ΔT_o will be equal to the temperature rise through the condensers only if the intake temperature is equal to T_a . Actual discharge temperatures should be relatively steady unless the power plant has several different condenser designs in which case the discharge temperature will vary with the plant load. Use of the theory requires that a constant value of ΔT_o be specified, the choice being based on the likely steady value of the discharge temperature.

The initial relative density difference, $\Delta \rho_o / \rho_a$ may be related to T_a and ΔT_o by $\Delta \rho_o / \rho_a = \beta \Delta T_o$ where β is a function of T as given in Figure 4.1 for fresh water. The curve of β vs. T is not linear but only a small error will be introduced if T is set equal to $T_a + \Delta T_o / 2$. A more accurate value of $\Delta \rho_o / \rho_a$ may be obtained by using tabulated values of water density vs. temperature, or in the case of salt water, water density vs. salinity and temperature.

The initial condenser water discharge velocity, u_o , is a function of the power plant condenser water pumping rate and the discharge channel area.

The discharge channel geometry is important since the theory uses the square root of one half of the discharge channel flow area as a scaling length. Calculation of the discharge densimetric Froude number, Fr_o , requires specification of the initial depth, h_o ; and the aspect ratio, A , requires both h_o and the initial width, b_o . The following procedure is suggested for any channel shape:

- a) Let h_o be the actual maximum discharge channel depth so that calculation of Fr_o is not affected by the schematization.
- b) Let b_o be such that the correct discharge channel area is preserved:

$$b_o = \frac{\text{channel area}}{2h_o} \quad (4.1)$$

Then the aspect ratio is given by:

$$A = \frac{h_o}{b_o} = \frac{2h_o^2}{\text{channel area}} \quad (4.2)$$

As an example, the aspect ratio of a circle is $8/\pi = 2.55$.

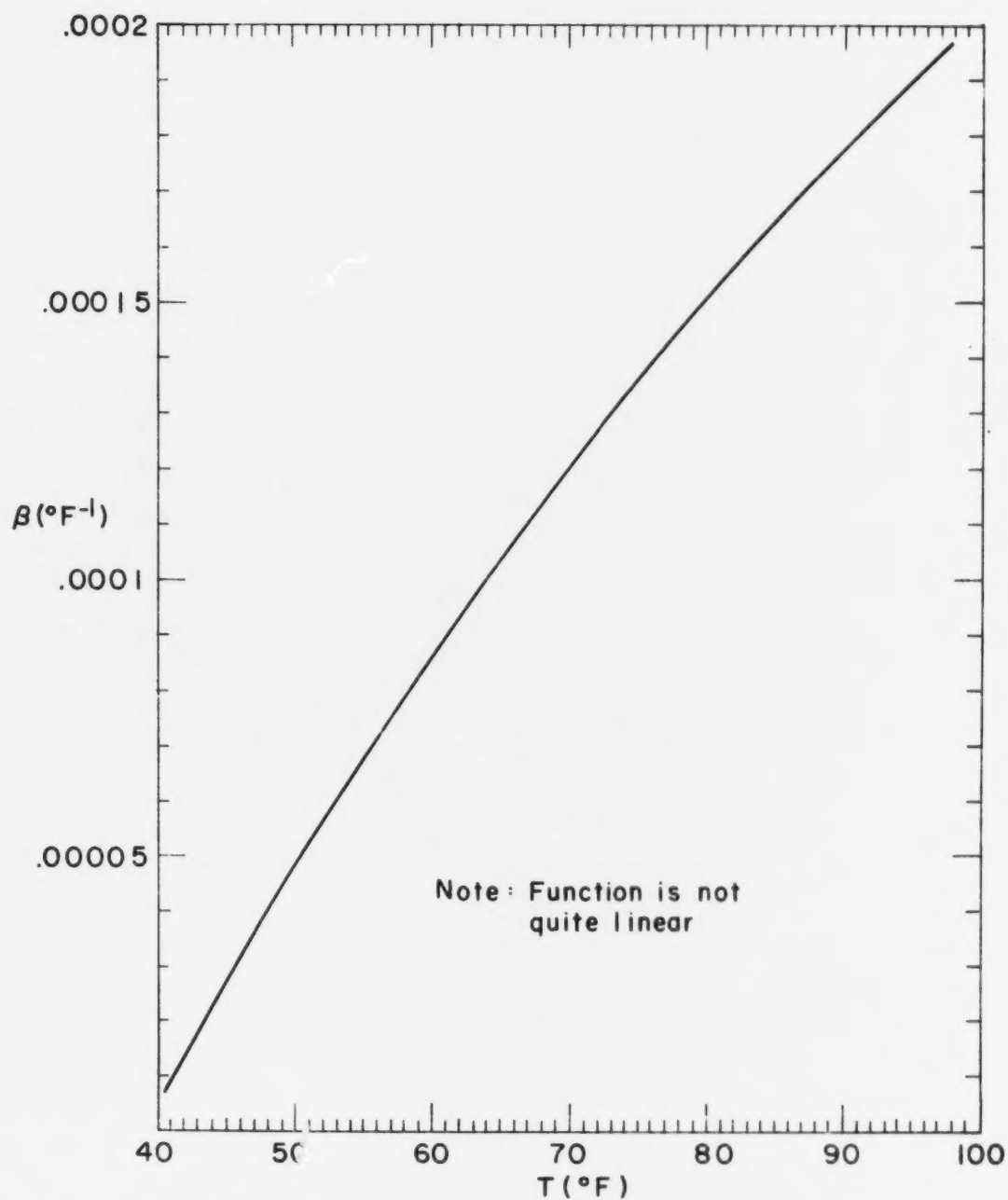


Fig. 4-1 Coefficient of Thermal Expansion for Fresh Water

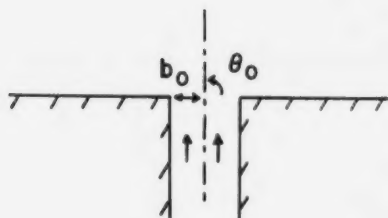
$$\beta = \frac{1}{\rho} \frac{\partial \rho}{\partial T} \quad (^{\circ}\text{F}^{-1}) \text{ as a Function of Temperature } T(^{\circ}\text{F})$$

The discharge channel geometry may vary with time if the elevation of the receiving water changes due to tidal motion or other causes. In this case separate calculations for each elevation of the receiving water must be made, assuming that the steady state theory predicts the instantaneous temperature distribution.

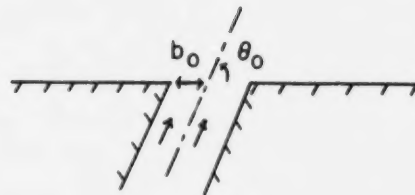
Very complicated arrangements of the discharge channel relative to the boundaries of the ambient receiving waters are beyond the capabilities of the theory of this study. If the discharge channel is nearly at right angles to the solid boundaries, the theory may be used as developed, or if the discharge is directed parallel to a straight boundary, the discharge may be schematized by assuming that the solid boundary is the centerline of a jet whose discharge channel is twice the width of the actual discharge (see Figure 4.2). The width b_o is then twice the width calculated by the procedure discussed previously. However, if it is not clear whether the jet will entrain water from one or both sides or if irregularly shaped solid boundaries will deflect the jet or distort it from the form assumed in the theory, a meaningful schematization is not possible.

The densimetric Froude number F_o and the aspect ratio A of the surface discharge channel should be chosen to be consistent with the bottom topography of the receiving water body. If optimum dilution is to be obtained, the vertical development of the surface jet should not be limited by the bottom of the receiving water. This is generally considered to be a desirable objective by the aquatic or marine biologist inasmuch as it avoids exposure of benthic organisms to high velocities and temperature rises. Designs which contain minimum interaction between the surface jet and the bottom topography are shown on the left side of Figure 4.3. Examples of surface jets in which there are substantial interactions in relation to the bottom topography are shown on the right side of Figure 4.3. In the latter case, vertical entrainment and dilution are reduced by the interference of the jet and the bottom.

If the discharge channel densimetric Froude number, F_o , is less than unity, a wedge of ambient water will intrude into the discharge channel and the heated flow will be forced to obtain a densimetric Froude number of unity at the discharge point (see Figure 4.4). The depth of the heated flow in the presence of a wedge, h_o^* , is given by:

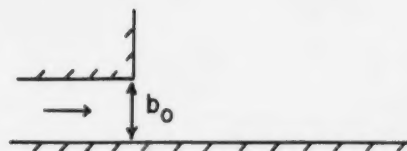
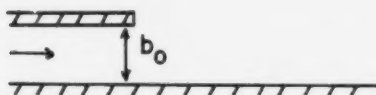


$$\theta_0 = 90^\circ$$

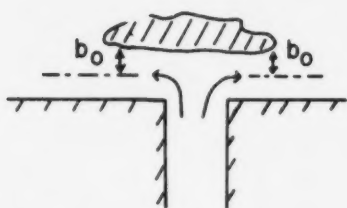


$$\theta_0 \text{ near } 90^\circ$$

Discharges with Entrainment from Both Sides



Discharges with Entrainment from One Side



Schematization Possible



Schematization not Possible
Because of Irregular Geometry

Discharges with Obstructions

Fig. 4.2 Discharge Channel Schematization

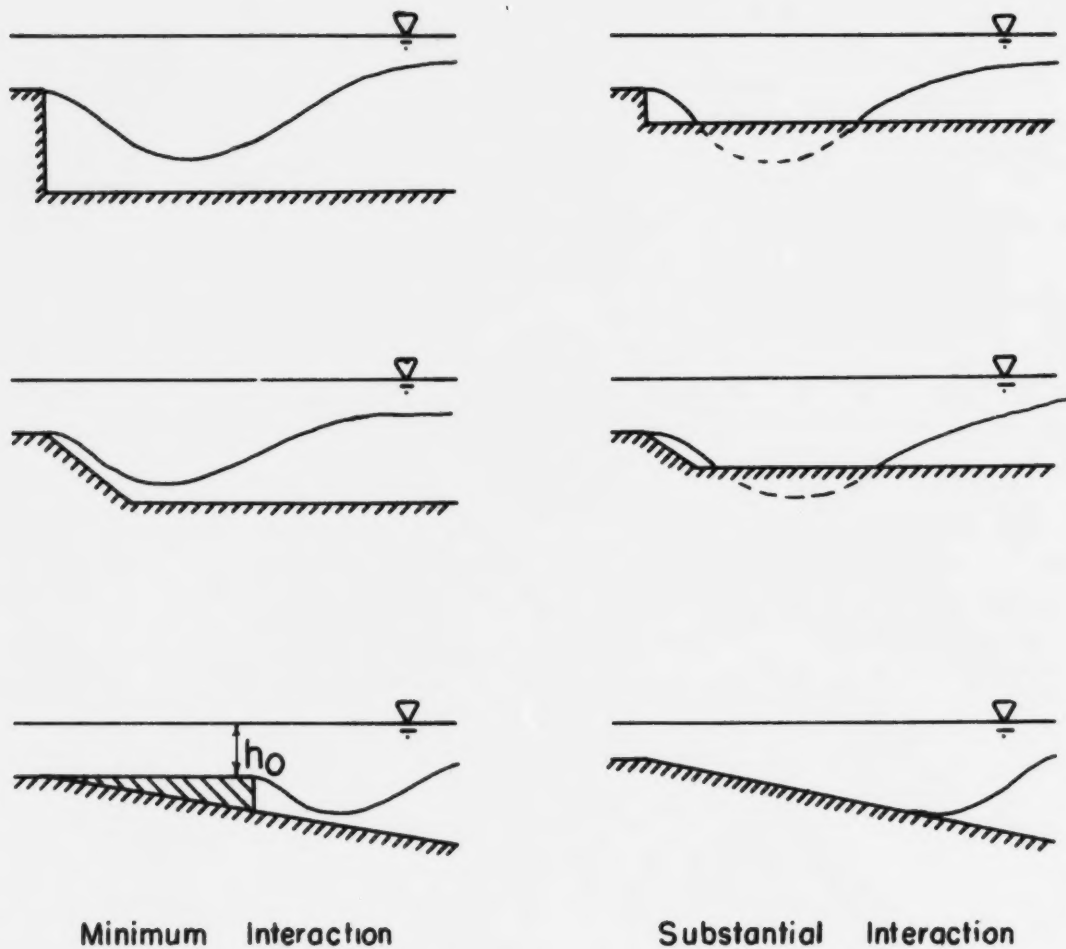


Fig. 4.3 Limitations on Maximum Jet Thickness by Bottom Topography

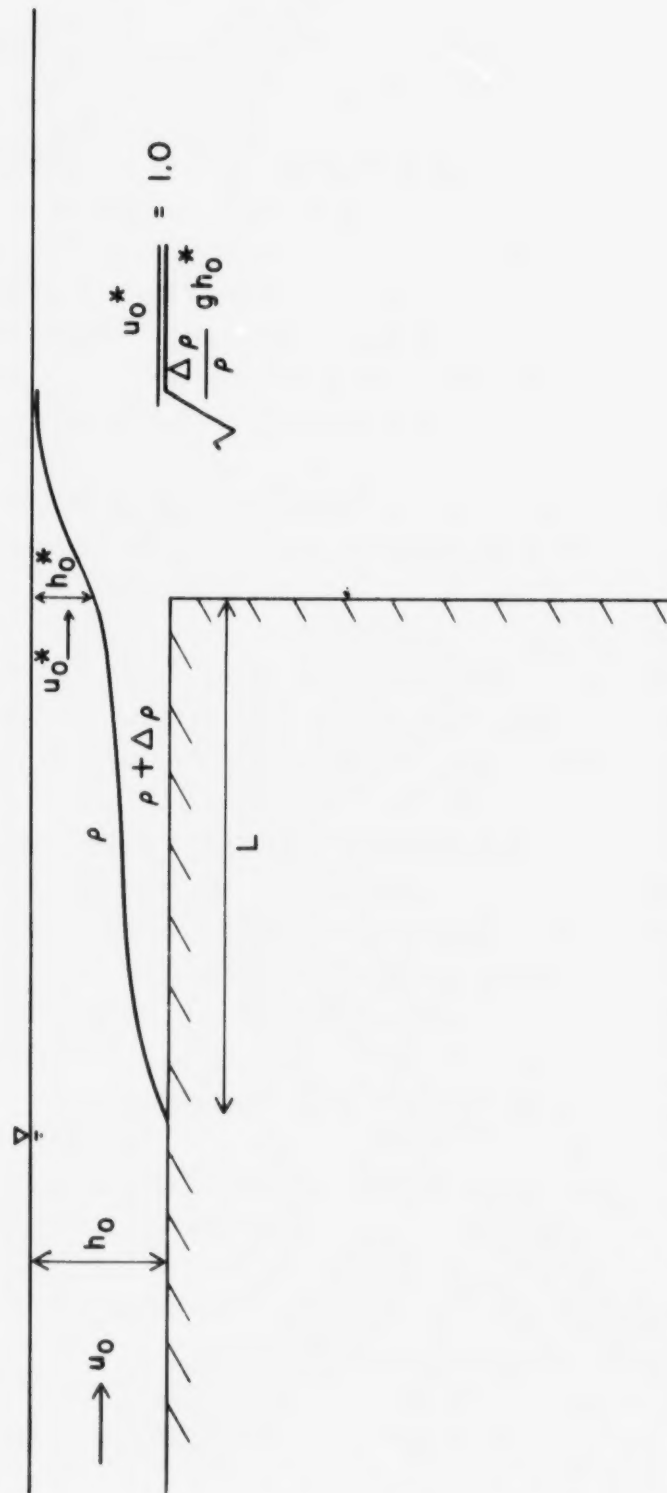


Fig. 4.4 Two Layer Flow in the Discharge Channel

$$\frac{h_o^*}{h_o} = F_o^{2/3} \quad (4.3)$$

where h_o and F_o are based on the channel dimensions. A flow area may be calculated based on the depth h_o^* and the aspect ratio calculated as described previously in this section.

The surface heat loss coefficient, k , may be estimated from local meteorological variables, principally wind speed, and the ambient water temperature. Care should be taken to use values for k which are appropriate for local conditions. Edinger (13) and Ryan (16) discuss methods for determining values of $K = \rho k$.

The cross flow velocity, V , in the receiving water may be measured directly or estimated from flow measurements in the case of a channel flow. The theory accepts as input values of V/u_o as a function of $\tilde{x}/\sqrt{h_o b_o}$ (see Chapter 3).

Once the schematization of the discharge configuration is achieved, the theoretical calculation is performed by the computer program described in Chapter 3. The inputs to the program for each calculation are F_o , $A = h_o/b_o$, k/u_o and V/u_o (as a function of $\tilde{x}/\sqrt{h_o b_o}$).

It should be noted that the theoretical model may not necessarily be applied to any arbitrary set of input parameters. In general the value of k/u_o has little effect; however, depending upon the value of the other parameters, F_o , A , and V/u_o , and upon the specified error size, ERR, the program may fail to generate a solution. This problem may take one of the following forms:

- 1) Little or no advancement of solution, i.e. the step size remains very small.
- 2) Abrupt discontinuities noticeable either in the values of the variables or in the total mass, momentum, or heat.
- 3) Unstable behavior in one or more variables, culminating usually in an overflow error.

The chance of the program encountering one of the above problems increases with decreasing F_o and A and increasing V/u_o . The value of ERR produces no consistent result except that too large a value of ERR most often yields

unstable behavior or discontinuities, and too small a value may prevent advancement.

The causes of this anomalous behavior in certain cases are not totally understood at the present time, but are thought to be related to the following:

- 1) accumulation of round-off error where the solution should be stable, i.e. a totally numerical problem,
- 2) genuinely unstable solutions, probably caused by the determinant of the differential equation coefficient matrix being zero or near zero.

The first of these could probably be solved by paying greater attention to round off error generation than is now done. To the extent that the second cause has no physical significance it may also be desirable to eliminate this problem by numerical manipulation. However, the governing equations for the heated discharge are similar to open channel flow equations which possess critical flow points at which the equations strictly have no solution and the coefficient matrix is zero. It is not clear to what extent the degeneracy of the three-dimensional heated discharge equations corresponds to critical flow behavior; the answer will only come by observing actual discharges in the laboratory and field. Thus for the time being the limitation on the computational range of this theory must be accepted.

Of course the values of F_0 , A , and V/u_0 are pre-determined by the case being considered, leaving ERR as the only free parameter. Figures 2.7a to 2.7i indicate the range of each variable and the various combinations of values for which calculations may be successfully performed. Table 4.1 gives a rough guideline as to the best value of ERR to be used for different cases. In general if the solution is not advancing, a larger ERR might be tried and if the solution appears unstable a smaller value should be tried. Experience has shown that the behavior of the solution may also depend upon the type of computer used. The results quoted herein are based on calculations done on an IBM 360 either -65 or -75. Users employing different systems than this are encouraged first of all to try several values of ERR in an attempt to make an unsuccessful calculation work and secondly to take up the challenge of illuminating more clearly than is done here the properties of the governing equations, the physical relevance of these properties, and their efficient numerical treatment.

Table 4.1

Guideline to Choice of Maximum Error Value, ERR

F_o	A	A	A	A
	0.1 - 0.5	0.5 - 1.0	1.0 - 1.5	2.0 - ∞
1 - 2	.005 with no crossflow	.005	.005	.005
	.05 with crossflow			
2 - 5	.005 with no crossflow	.005	.01	.01
	.05 with crossflow			
5 - 10	.01	.01	.01	.01
10 - ∞	.01	.01	.01	.01

4.2 Use of the Program Output

An example of the program output is shown in Appendix III. All of the outputs are in dimensionless form and must be transformed by the following steps

- 1) Multiply $\bar{x}$, $\bar{y}$, $\bar{x}$, $\bar{r}$, $\bar{s}$, $\bar{b}$, and $\bar{h}$ by $\sqrt{h_o b_o}$
- 2) Multiply $\bar{u}$ by u_o
- 3) Multiply $\bar{\Delta T}$ by ΔT
- 4) Multiply TM by $\sqrt{h_o b_o}/u_o$

The calculated isotherms may be constructed as follows:

- 1) Locate the centerline using $\bar{x}$ and $\bar{y}$.
- 2) Assign values of $T_c = T_a + \Delta T_c$ along the centerline.
- 3) Plot the boundaries of the core and turbulent regions using s , b , r and h .
- 4) Assign temperatures within the discharge using the assumed temperature distribution (Equation 2.5).

Figure 4.5 may be used as an aid in the construction of isotherm contours; lines of constant $\zeta_y = \frac{b+s}{b}$ or $\zeta_z = \frac{z-r}{h}$ are plotted and then the temperature contours are drawn by referring to the figure. The values of temperature rise above ambient are expressed as fractions of ΔT_o , the initial temperature rise.

If the surface area within a given temperature rise ΔT_* is of interest, the following formula may be used:

$$A_* = 2h_o b_o \int_0^{\bar{x}_*} \left[\bar{s} + \bar{b} \left(1 - \frac{\Delta T_*}{\Delta T} \right)^{2/3} \right] d\bar{x} \quad (4.4)$$

where

$$\left. \begin{aligned} \bar{s} &= s / \sqrt{h_o b_o} \\ \bar{b} &= b / \sqrt{h_o b_o} \\ \bar{\Delta T} &= \Delta T_c / \sqrt{h_o b_o} \end{aligned} \right\} \quad \text{As given in the program output as a function of } \bar{x}.$$

$$\bar{\Delta T}_* = \Delta T_* / \Delta T_o$$

$$\bar{x}_* = \text{the value of } x / \sqrt{h_o b_o} \text{ where } \bar{\Delta T} = \bar{\Delta T}_*$$

A simple computer program may be written which performs the above integration numerically.

Finally, the travel time along the centerline to a given temperature, $T_a + \Delta T_*$, is given by $TM(\bar{x}_*) \frac{\sqrt{h_o b_o}}{u_g}$.

It is important to note that although the theory calculates the temperature distribution out to very small values of temperature rise above ambient, these extreme regions of the discharge will be subject to distortion by random ambient processes especially wind stresses.

The calculated temperature distribution must be interpreted with this in mind and no critical significance should be given to the exact calculated position of isotherms of small temperature rise.

4.3 Case Study - Heated Surface Discharge into a Receiving Water Body of Finite Depth

Consider a proposed discharge into a large body of water at ambient temperature T_a with a uniform depth H as shown on Figure 4.6. A constraint is

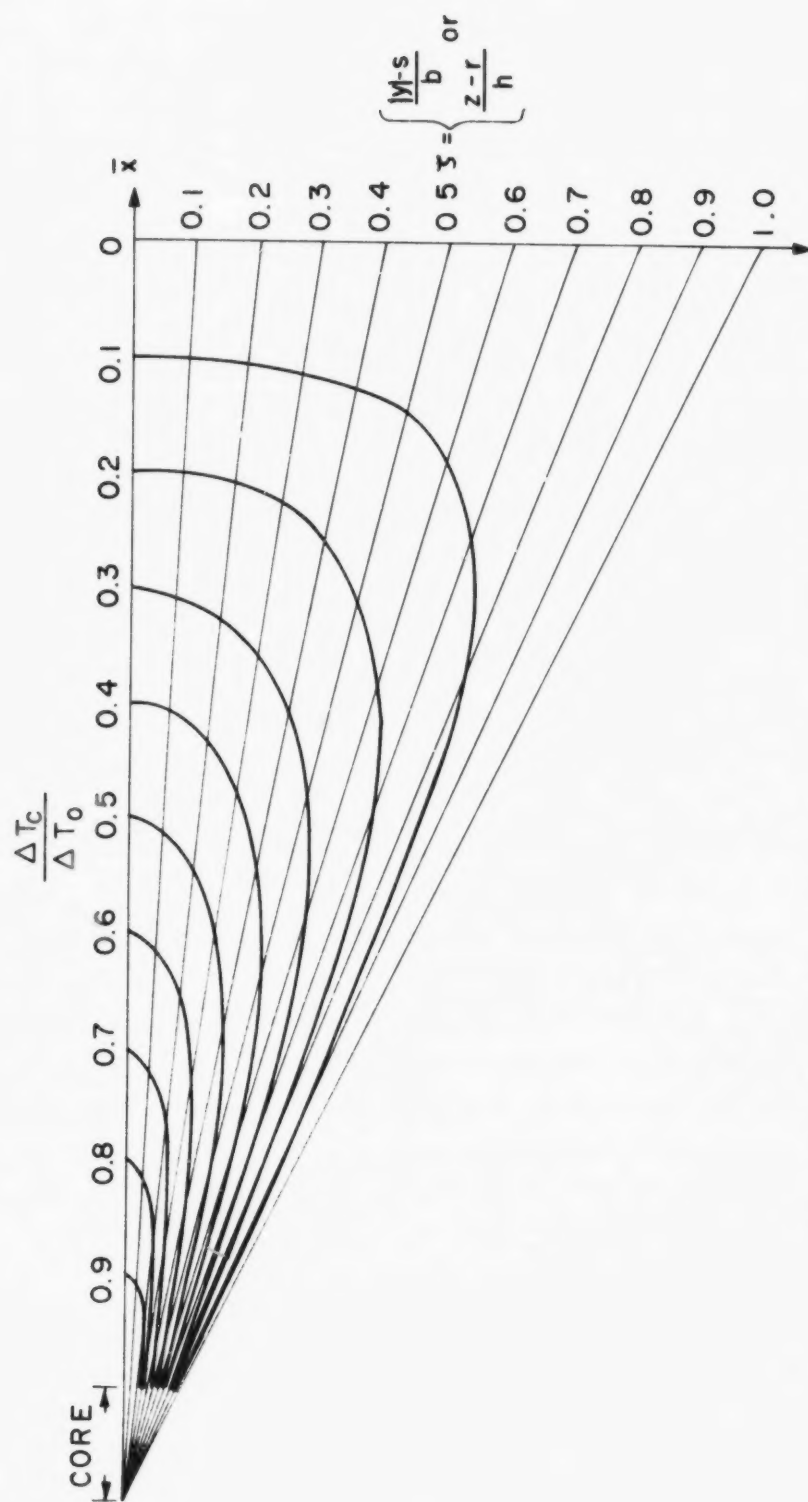


Fig. 4.5 Temperature Distribution Plotting Aid

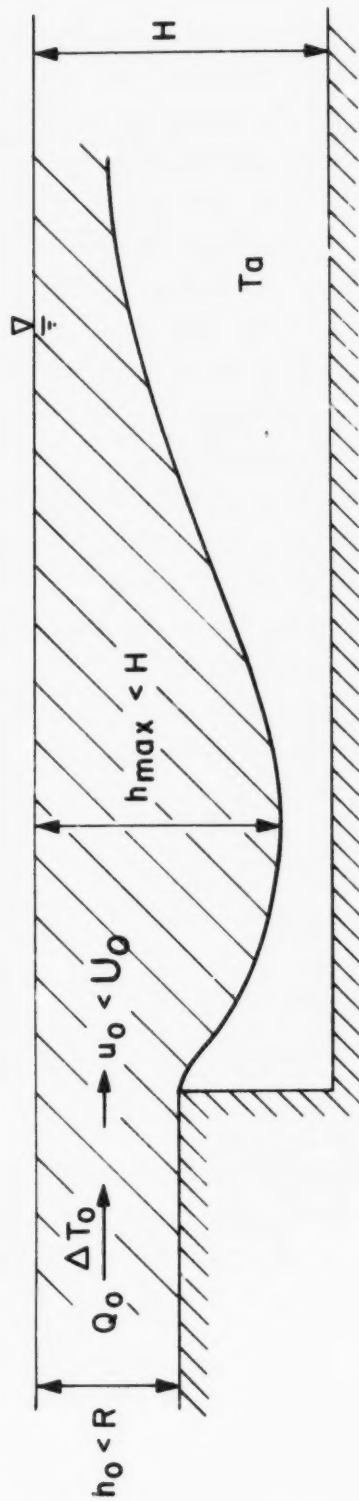


Fig. 4.6 Schematic of Case Study Problem

placed upon the discharge velocity such that $u_o \leq U_o$. Because the theory applies only to discharges which are not influenced significantly by solid boundaries in the receiving water, it is desirable that $h_{\max} \leq H$ where $h_{\max}$ is the maximum vertical penetration of the discharge (see Figure 4.6). The choice of $h_{\max}$ as the "critical" boundary of the jet is an arbitrary one but is probably conservative considering the assumed structure of the jet which sets $u = 0$ at that point. Entrainment should not be significantly affected by contact with the bottom at the point of maximum depth only.

A third constraint may be imposed by local topography. For instance h_o must be less than or equal to the water depth, and $2b_o$ may be constrained by some critical width. Thus in general $h_o < R$ and $b_o < S$.

The discharge is characterized by an initial temperature rise, ΔT_o , and a condenser water flow, Q_o . The problem treated in this case study is to design a discharge channel for maximum ultimate (stable) dilution (see Figure 2.5) while meeting all the imposed constraints. This case study will illustrate the generation of temperature contours and isotherm areas from the computer output in addition to treating the more specific, but often relevant, problem of optimizing a channel design with one or more constraints imposed.

The definition of IF'_o and the following approximate formulas may be recalled from Chapter 2 for IF'_o greater than 3.

$$IF'_o = IF_o A^{1/4} = \frac{u_o}{(g\beta\Delta T_o)^{1/2} (h_o b_o)^{1/4}} \quad (4.5a)$$

$$= \frac{Q_o}{2(g\beta\Delta T_o)^{1/2} (h_o b_o)^{5/4}} \quad (4.5b)$$

$$= \frac{2^{1/4} u_o^{5/4}}{(g\beta\Delta T_o)^{1/2} Q_o^{1/4}} \quad (4.5c)$$

$$\left(\frac{\Delta T_o}{\Delta T_{c_s}} \right) \approx IF'_o \quad (4.6a)$$

$$D_s \approx 1.4 F'_0 \quad (4.6b)$$

$$\frac{h_{\max}}{(h_0 b_0)^{1/2}} \approx 0.42 F'_0 \quad (4.7)$$

Equation (4.5) defines F'_0 , (4.6a and b) relates centerline dilution and overall dilution to F'_0 and (4.7) relates the maximum jet penetration to F'_0 . Note that for a given $\Delta\rho_0/\rho$ and Q_0 both the maximum stable dilution and the maximum penetration are dependent only on discharge velocity u_0 and not on the aspect ratio. This provides freedom in selecting channel geometry.

The constraint that $h_m \leq H$ may be restated using (4-7) as

$$F'_0 \leq \frac{6.8 H^{5/3} (g\beta\Delta T_0)^{1/3}}{Q_0^{2/3}} \quad (4.8)$$

and from (4.5c) the constraint that $u_0 \leq U_0$ may be restated as

$$F'_0 \leq \frac{1.19 U_0^{5/4}}{(g\beta\Delta T_0)^{1/2} Q_0^{1/4}} \quad (4.9)$$

Because ultimate dilution increases monotonically with F'_0 (see Eqn. 4.6), the channel should be designed with F'_0 equal to the smaller of the two expressions in (4.8) and (4.9). From (4.5b) it may be shown that

$$h_0 b_0 = \frac{Q_0^{4/5}}{2^{4/5} F'^{4/5}_0 (g\beta\Delta T_0)^{2/5}} \quad \text{and} \quad (4.10)$$

$$u_0 = \frac{Q_0}{2h_0 b_0} \quad (4.11)$$

where F'_0 is picked from (4.8 and 4.9). If there is a constraint on either the width or depth of the discharge structure, this constraint may be included with (4.10) to determine h_0 and b_0 . If there is a constraint on both width and depth

such that $h_o < R$ and $b_o < S$ and the product $RS < h_o b_o$ as determined by (4.10), then a discharge channel cannot be designed with the desired constraints.

Example: Design a discharge channel no more than 12 feet deep to achieve maximum stable dilution of a heated discharge of 2,000 cfs and 15°F temperature rise into a water body 31 ft. deep at an ambient temperature of 70°F. Maximum discharge velocity is $U_o = 6$ ft/sec. The area within the 4°F surface isotherm should be calculated.

1. The value of β is taken from Figure 4-1 using $T = 77.5^\circ\text{F}$, $\beta = .000144^\circ\text{F}^{-1}$ and $(g\beta\Delta T_o) = .069 \text{ ft/sec}^2$.
2. Using (4.8) the constraint on depth is

$$F_o' \leq \frac{6.8(31)^{5/3}(.069)^{1/3}}{2000^{2/3}} = 5.3$$

Using (4.9) the constraint on velocity is

$$F_o' \leq \frac{(1.19)(6)^{5/4}}{(.069)^{1/2}(2000)^{1/4}} = 6.4$$

Hence F_o' should equal 5.3 and the jet should just touch the bottom.

3. From (4.10) and (4.11)

$$h_o b_o = \frac{2000^{4/5}}{2^{4/5}(5.3)^{4/5}(.069)^{2/5}} = 192 \text{ ft}^2,$$

$$(h_o b_o)^{1/2} = \text{scaling length} = 13.9 \text{ ft.}$$

$$\text{and } u_o = \frac{2000}{2(192)} = 5.2 \text{ ft/sec}$$

4. No constraint has been placed on b_o so any number of combinations of $h_o b_o$ will be satisfactory. As an example, $h_o = 11$ ft. and $b_o = 17.5$ feet satisfies the requirement on channel depth and results in a $F_o' = 6.0$; aspect ratio, $A \approx 0.6$; and $F_o' = 5.3$.

5. Theoretical calculation (computer output) for these values are shown in Appendix III.
6. Figure 4.5 is used to plot the isotherms in two planes (horizontal at the surface and vertical at the jet centerline) in Figure 4.7.
7. Equation 4.4 is used to calculate the area within the 4° isotherm with $\overline{\Delta T}_* = 4/15 = .27$ and $\bar{x}_* = 45$. The computations are organized as follows:

$\bar{x}$	$\bar{s}$	$\bar{b}$	$\overline{\Delta T}$	$\Delta T_*/\Delta T$	$(1-\overline{\Delta T}_*/\overline{\Delta T})^{2/3}$	$\bar{s}+\bar{b}(1-\overline{\Delta T}_*/\overline{\Delta T})^{2/3}$	$\sum [\bar{s}+\bar{b}(1-\overline{\Delta T}_*/\overline{\Delta T})^{2/3}] \Delta \bar{x}$
1.131	1.52	.585	.971	.278	.805	1.99	2.25
2.62	1.37	1.16	.940	.287	.798	2.30	5.67
5.24	1.44	2.39	.884	.305	.784	3.31	14.3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
10.5	1.48	5.27	.692	.390	.719	5.27	39.3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
41.9	0	41.5	.282	.957	.122	5.01	267.

The first and last column may be multiplied by the scaling length, $\sqrt{h_o b_o} = 13.9$ ft. and plotted against each other to produce the 74° isotherm shown in Fig. 4.7. The integration in Equation 4.4 results in an $A_* = 2(192) (267) = 103,000$ sq. ft. or 2.4 acres.

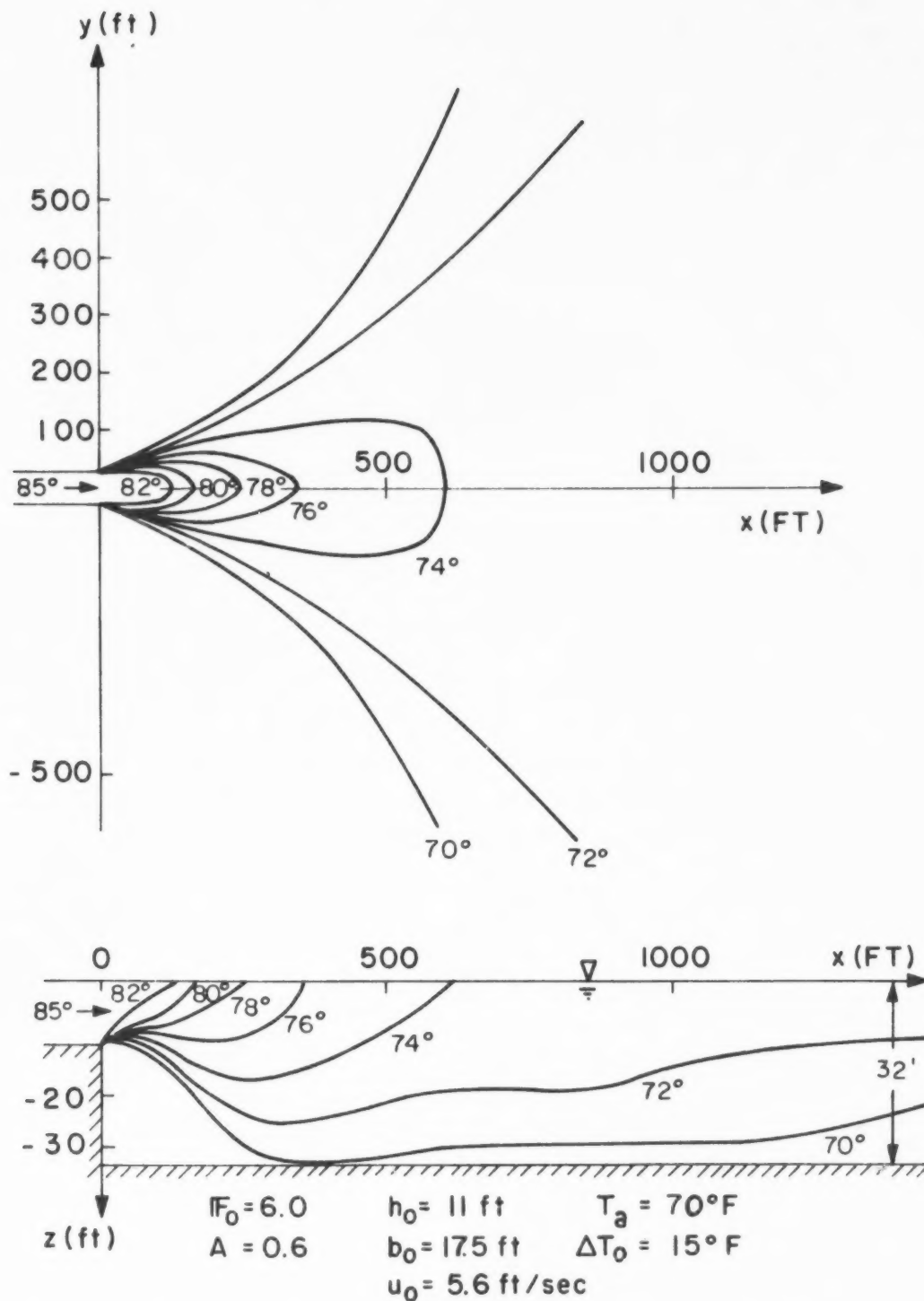


Fig. 4.7 Calculated Contours for the Case Study Example

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List of Symbols

- A - discharge channel aspect ratio, h_o/b_o
- a_{ij} - coefficient matrix in the governing differential equation set
- b - horizontal surface distance from core boundary to jet boundary
- b_o - one half the width of rectangular discharge channel
- C - coefficient in the exponent of Ellison and Turner's vertical entrainment velocity function
- c_{ij} - vector of the constants in the governing differential equation jet
- D - ratio of flow in the jet to the initial flow = dilution
- D_s - dilution in the stable region of the heated discharge
- $\frac{db}{dx}$ - lateral spread of the turbulent region of a jet
- ERR - maximum allowable average roundoff error for each step in the numerical computation
- IF_o - densimetric Froude number of the discharge channel = $\frac{u_o}{\sqrt{\frac{\Delta\rho_o}{\Delta\rho} gh_o}}$
- IF_L - local densimetric Froude number in the jet = $\frac{u_c}{\sqrt{\frac{\Delta\rho}{\Delta\rho} gh}}$
- IF_o' - a characteristic Froude number = $IF_o A^{1/4}$
- f - similarity function for velocity = $(1 - \zeta^{3/2})^2$
- g - acceleration of gravity
- HT - ratio of heat flow in the jet to the initial discharge heat flow
- H - maximum allowable vertical penetration of the jet
- h - vertical centerline distance from core boundary to jet boundary
- h_o - depth of the discharge channel
- h_o^* - depth of the heated flow at the point of discharge if a cold water wedge is present
- h_{max} - maximum value of h obtained in a heated discharge
- I_1 - $\int_0^1 f(\zeta) d\zeta = .4500$
- I_2 - $\int_0^1 f^2(\zeta) d\zeta = .3600$
- I_3 - $\int_0^1 t(\zeta) d\zeta = .6000$

$$I_4 - \int_0^1 \int_{\zeta}^1 t(\zeta) d\zeta d\zeta = .2143$$

$$I_5 - \int_0^1 f(\zeta) \zeta^{1/2} d\zeta = .2222$$

$$I_6 - \int_0^1 f^2(\zeta) \zeta^{1/2} d\zeta = .1333$$

$$I_7 - \int_0^1 f(\zeta) t(\zeta) d\zeta = .3680$$

- K - surface heat loss coefficient
- k - kinematic surface heat loss coefficient
- P - pressure
- Q_o - discharge channel flow
- R - maximum allowable depth of discharge channel
- r - vertical distance from the jet centerline to the boundary of the core region
- S - maximum allowable half-width of discharge channel
- s - horizontal distance from the jet centerline to the boundary of the core region
- STEP - increment in $x/\sqrt{h_o b_o}$ at which numerical output is printed
- T - temperature
- T_a - ambient temperature
- T_c - jet centerline surface temperature
- T_o - temperature of the heated flow in the discharge channel
- T_{*} - a particular temperature of interest
- ΔT - temperature rise above ambient in the jet, $T - T_a$
- ΔT_c - surface temperature rise above ambient at the jet centerline, $T_c - T_a$
- ΔT_o - temperature difference between the discharge and the ambient water,

$$\frac{T_o - T_a}{T_* - T_a}$$
- ΔT_{*} - $T_* - T_a$
- TM - dimensionless time of travel along the jet centerline,

$$\frac{u_o}{\sqrt{h_o b_o}} \int_0^x \frac{dx}{u_c} .$$
- t - similarity function for temperature = $(1-\zeta)^{3/2}$

- u, v, w - velocity components in the coordinate system relative to the centerline of a deflected jet
- $\tilde{u}, \tilde{v}, \tilde{w}$ - velocity components in the fixed coordinate system
- U_o - maximum allowable velocity in the discharge channel
- u_o - velocity in the discharge channel
- u_o^* - velocity of the heated flow at the point of discharge if a cold water wedge is present
- u_c - surface centerline jet velocity
- V - ambient crossflow velocity
- $V_1 - V_8$ - ambient crossflow velocity components as input to the numerical program
- v_e - lateral velocity of the entrained flow at the jet boundary
- v_c - lateral velocity in the jet at $y = s+b$ and $-(h+r) < z < \eta$
- v_h - lateral velocity in the jet at $y = s$ and $-(h+r) < z < -r$
- v_s - lateral velocity in the jet at $y = s$ and $-r < z < \eta$
- w_c - vertical velocity in the jet at $z = -(r+h)$ and $0 < y < s+b$
- w_b - vertical velocity in the jet at $z = -r$ and $b < y < s+b$
- w_e - vertical velocity in the jet at $z = -r$ and $0 < y < s$
- $\tilde{x}, \tilde{y}, \tilde{z}$ - coordinate direction relative to the centerline of a deflected jet
- $\tilde{x}, \tilde{y}, \tilde{z}$ - fixed coordinate direction
- x_* - value of x where $\Delta T = \Delta T_*$
- x_L - value of $x/\sqrt{h_o b_o}$ at which the numerical computation terminates
- y_j - vector of the variables ($\tilde{u}, \Delta T$, etc.) in the governing differential equation set

- α_y - lateral entrainment coefficient in non-buoyant and buoyant jets
- α_z - vertical entrainment coefficient in a non-buoyant jet
- α_{sz} - vertical entrainment coefficient in a buoyant jet
- β - coefficient of thermal expansion of water
- ϵ - spread, $\frac{db}{dx}$, of the turbulent region of a non-buoyant jet
- ϵ_o - spread, $\frac{db}{dx}$, of the turbulent region in an undeflected non-buoyant jet
- ζ_y - dimensionless width of the turbulent region of a jet, $\frac{|y|-s}{b}$
- ζ_z - dimensionless depth of the turbulent region of a jet, $\frac{z-r}{h}$

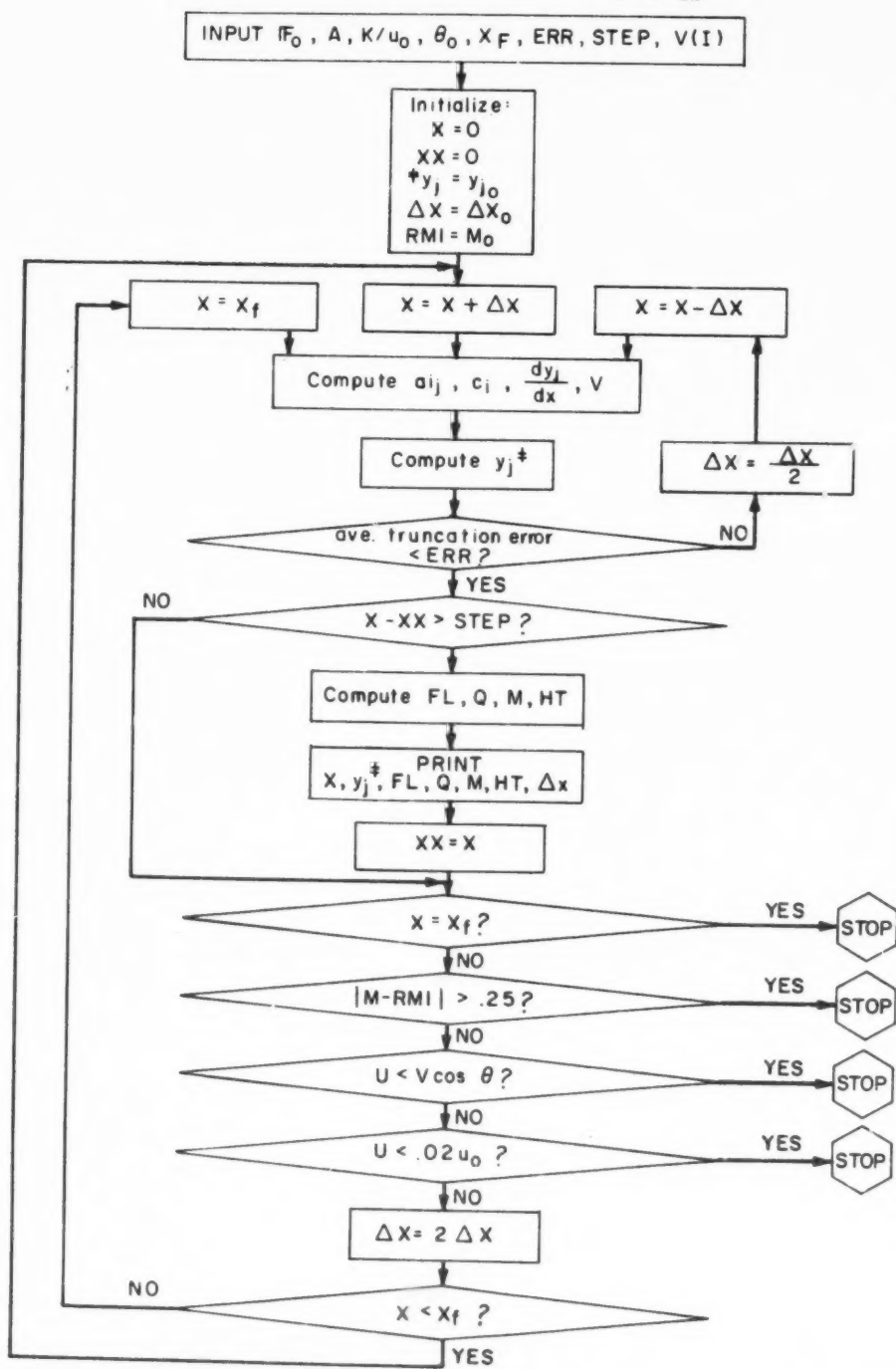
- ζ - either of ζ_y or ζ_z
- θ - angle between the jet centerline (x axis) and the $\tilde{y}$ axis
- θ_o - angle between the discharge channel centerline and the $\tilde{y}$ axis
- ρ - density of water
- ρ_a - density of the ambient water
- ρ_o - density of the heated discharge
- $\Delta\rho$ - difference between the ambient water density and the water density, $\rho_a - \rho$
- $\Delta\rho_o$ - difference between the ambient water density and the density of
the heated flow in the discharge channel, $\rho_a - \rho_o$

superscript ' - indicates a turbulent fluctuating quantity

superscript - - indicates a dimensionless quantity. Note that variables printed by the computer are dimensionless but are capitalized.

subscript s - indicates a jet property in the stable region

Appendix I Flow Chart for Solution of $[a_{ij}] \frac{dy_j}{dx} = c_i$



* y_j include U, T, B, H, R, S, $\frac{dB}{dx}$, θ , XP, and YP; $\frac{dB}{dx}$ is not printed

```

DOUBLE PRECISION DERV,Y,PKMT,X,TH,XP,YP,U,B,T,F,BX,S,R,XL14,AUX,XX
DOUBLE PRECISION A,C,CCUS,DSIN,DELX
REAL I1,I2,I3,I4,I5,I6,I7,I8,M
INTEGER P,RR,REG,VFG,FLAG
DIMENSION PKMT(5),Y(10),AUX(5,10),DERV(10),A(10,10),C(10),VEL(3)
EQUIVALENCE (Y(5),TH),(Y(9),XP),(Y(10),YP),(Y(3),B),(Y(1),U)
EQUIVALENCE (Y(2),T),(Y(4),H),(Y(7),BX),(Y(5),S),(Y(5),R)
COMMON DERY,Y,PKMT,X,XLIM,AUX,XX,A,C,DELX
COMMON EPS8,G,DV,IFR,VIR,M,VI,KMI,M,FR,AS,E
COMMON THIO,THI,CRK,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,DT,FI
COMMON P,RR,NPAGE,NLINE,REG,NDIM,VFG,FLAG,IHLF,STEP,IX,XI
EPS=0.22
I1=.4500
I2=.3160
I3=.6000
I4=.2143
I5=.2220
I6=.1333
I7=.3080
RR=5
100 READ (RR,100) KK
    FORMAT (I3)
    K=1
    10 READ(RR,101) FR,AS,L,THIO,XLIM,ERR,STEP
    101 FORMAT (2F10.5,F10.7,4F10.5)
    102 READ (RR,102) (VEL(I),I=1,5)
    102 FORMAT (8F10.5)
    THI=3.14159*THIO/180.0
    R=AS**5
    S=1./R
    F=.0001
    XX=-2.0
    D=H
    T=1.0
    BX=EPS
    TH=THI

```

Appendix II Program Listing

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```

XP=0.
YP=0.
CALL CRUSS
U=1.0-V*COS(THI)
IER=0
TM=0
XI=0
NPAGE=0
NLINE=50
REG=1
FI=(1.0/(FR**2))*S
RMI=1.0+R*FI*.5
PRMT(1)=0.
PRMT(2)=XLIM
PRMT(3)=.00001
PRMT(4)=ERR
IF (VEL(1)) 2,1,2
1 NDIM=7
VFG=2
GO TO 3
2 NDIM=10
VFG=1
3 DO 4 N=1,NDIM
DERY(N)=1./NDIM
4 CONTINUE
CALL SRKGS
IF(K-KK) 7,8,8
7 K=K+1
GO TO 10
8 CALL EXIT
9 STOP
END

```

```

PGM10037
PGM10038
PGM10039
PGM10040
PGM10041
PGM10042
PGM10043
PGM10044
PGM10045
PGM10046
PGM10047
PGM10048
PGM10049
PGM10050
PGM10051
PGM10052
PGM10053
PGM10054
PGM10055
PGM10056
PGM10057
PGM10058
PGM10059
PGM10060
PGM10061
PGM10062
PGM10063
PGM10064
PGM10065
PGM10066
PGM10067
PGM10068

```



```

C      PREPARATIONS OF FIRST RUNGE-KUTTA STEP
      DO 3 I=1,NDIM
        AUX(1,I)=Y(I)
        AUX(2,I)=DERV(I)
        AUX(3,I)=0.DC
        AUX(4,I)=0.DC
      3   IREC=0
        H=H+H
        IHLF=-1
        ISTEP=0
        IEND=0

C      C
C      4   IF((X*H-XEND)*H) 7,6,5
C      C
C      START OF A RUNGE-KUTTA STEP
      5   H=XEND-X
      6   IEND=1

C      C
C      RECORDING OF INITIAL VALUES OF THIS STEP
      7   CALL OUTP
      36  IF(PRMT(5))40,8,40
      8   ITEST=0
      9   ISTEP=ISTEP+1

C      C
C      START OF INNERMOST RUNGE-KUTTA LOOP
      J=1
      DO 91 I=1,NDIM
        AUX(3,I)=0.DC
        91  AUX(6,I)=0.DC
      10  AJ=A(J)
        BJ=B(J)
        CJ=C(J)
        DO 11 I=1,NDIM
          R1=H*DERV(I)
          R2=AJ*(R1-BJ*AUX(6,I))

```

```

SRKG0037
SRKG0038
SRKG0039
SRKG0040
SRKG0041
SRKG0042
SRKG0043
SRKG0044
SRKG0045
SRKG0046
SRKG0047
SRKG0048
SRKG0049
SRKG0050
SRKG0051
SRKG0052
SRKG0053
SRKG0054
SRKG0055
SRKG0056
SRKG0057
SRKG0058
SRKG0059
SRKG0060
SRKG0061
SRKG0062
SRKG0063
SRKG0064
SRKG0065
SRKG0066
SRKG0067
SRKG0068
SRKG0069
SRKG0070
SRKG0071
SRKG0072

```

```

V(I)=Y(I)+R2
R2=R2+R2+R2
11 AUX(6,I)=AUX(6,I)+R2-CJ#R1
IF(J-4)12,15,15
12 J=J+1
IF(J-3)13,14,13
13 X=X+.500*H
14 CALL FCT
GO TO 10
END OF INNERMOST RUNGE-KUTTA LOOP
C
C
TEST OF ACCURACY
15 IF(ITEST)16,16,20
C
IN CASE ITEST=0 TESTING OF ACCURACY IS IMPOSSIBLE
16 DO 17 I=1,NDIM
17 AUX(4,I)=Y(I)
ITEST=1
ISTEP=ISTEP+ISTEP-2
18 IHLF=IHLF+1
X=X-H
H=.500*H
DO 19 I=1,NDIM
Y(I)=AUX(I,I)
DERY(I)=AUX(2,I)
19 AUX(4,I)=AUX(3,I)
GO TO 9
C
C
IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE
20 IMDD=ISTEP/2
IF(ISTEP-IMDD-IMDD)21,23,21
21 CALL FCT
DO 22 I=1,NDIM
AUX(5,I)=Y(I)
22 AUX(7,I)=DERY(I)
GO TO 9
C
C

```

SRKG0073
SRKG0074
SRKG0075
SRKG0076
SRKG0077
SRKG0078
SRKG0079
SRKG0080
SRKG0081
SRKG0082
SRKG0083
SRKG0084
SRKG0085
SRKG0086
SRKG0087
SRKG0088
SRKG0089
SRKG0090
SRKG0091
SRKG0092
SRKG0093
SRKG0094
SRKG0095
SRKG0096
SRKG0097
SRKG0098
SRKG0099
SRKG0100
SRKG0101
SRKG0102
SRKG0103
SRKG0104
SRKG0105
SRKG0106
SRKG0107
SRKG0108

```

C
C      COMPUTATION OF TEST VALUE DELT
23 DELT=0.00
   DO 24 I=1,NDIM
24 DELT=DELT+AUX(5,I)*DAES(AUX(4,I)-Y(I))
   IF (DELT-PRMT(4))23,28,25
C
C      ERROR IS TOO GREAT
25 GO TO 26
26 DO 27 I=1,NDIM
27 AUX(4,I)=AUX(5,I)
   ISTEP=ISTEP+ISTEP-4
   X=X-H
   IEND=0
   GO TO 13
C
C      RESULT VALUES ARE GUCC
23 DO 50 K=1,6
   IF (Y(K)) 25,50,50
50 CONTINUE
281 CALL FCT
   DO 29 I=1,NDIM
   AUX(1,I)=Y(I)
   AUX(2,I)=DERY(I)
   AUX(3,I)=AUX(6,I)
   Y(I)=AUX(5,I)
29 DERY(I)=AUX(7,I)
   IF (PRMT(5))40,30,40
30 DO 31 I=1,NDIM
   Y(I)=AUX(1,I)
31 DERY(I)=AUX(2,I)
   IREC=IHLF
   IF (IEND)32,32,39
C
C      INCREMENT GETS DOUBLED
32 IHLF=IHLF-1

```

```

SRKG0139
SRKG0110
SRKG0111
SRKG0112
SRKG0113
SRKG0114
SRKG0115
SRKG0116
SRKG0117
SRKG0118
SRKG0119
SRKG0120
SRKG0121
SRKG0122
SRKG0123
SRKG0124
SRKG0125
SRKG0126
SRKG0127
SRKG0128
SRKG0129
SRKG0130
SRKG0131
SRKG0132
SRKG0133
SRKG0134
SRKG0135
SRKG0136
SRKG0137
SRKG0138
SRKG0139
SRKG0140
SRKG0141
SRKG0142
SRKG0143
SRKG0144

```

```

ISTEP=ISTEP/2
H=H+H
33 IMCD=ISTEP/2
  IF (ISTEP-IMCD-IMOD) 4, 24, 4
34 GO TO 35
35 IHLF=IHLF-1
  ISTEP=ISTEP/2
  H=H+H
  GO TO 4
C
C
  RETURNS TO CALLING PROGRAM
37 IHLF=12
  CU TO 39
38 IHLF=13
39 CALL OUTP
40 RETURN
END

```

```

SRKG0145
SRKG0146
SRKG0147
SRKG0148
SRKG0149
SRKG0150
SRKG0151
SRKG0152
SRKG0153
SRKG0154
SRKG0155
SRKG0156
SRKG0157
SRKG0158
SRKG0159
SRKG0160
SRKG0161

```



```

2021 FORMAT(//2X,13HERCULE NUMBER,6X,12WASPCT RATIO,7X,
118HANGLE OF DISCHARGE,6X,14HRLUND.FF ERROR,7X,4HXL14,14X,
214HFEAT LOSS COEF)
WRITE (P,203) FR,AS,THU,ER,XLIN,F
203 FORMAT(1X,4(F9.5,12X),F10.5,11X,FC.7//)
WRITE (P,300) (I,VEL(I),I=1,3)
300 FORMAT(3(3X,3HVEL,I1,1H=,F6.3)//)
WRITE (P,159)
159 FORMAT(4X,1HX,6X,1HH,6X,1HB,6X,1HR,3X,1HS,6X,2HFL,7X,
11H0,7X,1HM,5X,1HU,5X,1HT,6X,2HHT,5X,1HV,6X,2HHP,
27X,2HYP,5X,3HTHU,6X,2HTM//)
150 NLIN=NLIN+1
THD=180.0*TH/3.14159
FL=T*H/(U*U)
FL=FL/(AS**.5)
FL=ABS(FL)
FL=FL**.5
FL=FR/FL
Q=U*(S+B*I1)*(R+H*I1)+V*DCUS(TH)*(S+B)*(R+H)
M=U*U*(S+B*I2)*(R+H*I2)+Q*V*DCUS(TH)
M=M+U*V*DCUS(TH)*(S+B*I1)*(R+H*I1)
M=M+(R*R**.5+I3*H*R+I4*H*H)*(S+B*I3)*I*FI
HT=U*HT*(S+B*I7)*(R+H*I7)+I*V*DCUS(TH)*(S+B*I3)*(R+H*I3)
DELX=X-XI
DTM=2*DELX/(U+UI)
TM=TM+DTM
95 IF(NDIM-7)96,96,97
96 XP=X
97 CONTINUE
WRITE (P,200) X,H,B,R,S,FL,Q,M,I1,HT,V,XP,YP,THD,TM
200 FORMAT(7(1X,D8.3),5(1X,F9.3),2(1X,D3.3),1X,F5.1,2X,D3.3)
XI=X
UI=U
IF(ABS(M-RMI)-.25) 160,160,159
160 IF(U-V*DCUS(TH))160,160,159
159 IF(U-.02) 160,160,170

```

```

001P0037
001P0038
001P0039
001P0040
001P0041
001P0042
001P0043
001P0044
001P0045
001P0046
001P0047
001P0048
001P0049
001P0050
001P0051
001P0052
001P0053
001P0054
001P0055
001P0056
001P0057
001P0058
001P0059
001P0060
001P0061
001P0062
001P0063
001P0064
001P0065
001P0066
001P0067
001P0068
001P0069
001P0070
001P0071
001P0072

```

```

158 WRITE (P,400)
400 FORMAT(/10X,43HMOMENTUM HAS EXCEEDED BOUNDS RUN TERMINATES)
    GO TO 190
160 WRITE (P,401)
401 FORMAT(/10X,48HJET VELOCITY HAS BECOME TOO SMALL RUN TERMINATES)
190 PRMT(5)=1.0
170 RETURN
    END

```

```

OUTP0073
OUTP0074
OUTP0075
OUTP0076
OUTP0077
OUTP0078
OUTP0079
OUTP0080

```

```

SUBROUTINE CRUSS
DOUBLE PRECISION DERY,Y,PRMT,X,TH,AP,YP,L,B,T,F,BX,S,R,XLIM,AUA,XX
DOUBLE PRECISION AUN,CUN,DCIS,DSIM,DELX
REAL I1,I2,I3,I4,I5,I6,I7,I8,M
INTEGER P,RR,REG,VFG,FLAG
DIMENSION PRMT(5),Y(10),AUA(6,10),DERY(10),VEL(6)
DIMENSION AUN(10,10),CUN(10)
EQUIVALENCE (Y(8),TH),(Y(9),XP),(Y(10),YP),(Y(3),B),(Y(1),U)
EQUIVALENCE (Y(2),T),(Y(4),H),(Y(7),BX),(Y(6),S),(Y(5),R)
COMMON DERY,Y,PRMT,X,XLIM,AUX,XX,AUN,CUN,DELX
COMMON EPSH,G,GV,IER,IR,WI,V1,KMI,M,FA,AS,E
COMMON THIO,THI,ERR,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,LT,FI
COMMON P,RR,NPAGE,NLINE,REG,NJIM,VFG,FLAG,IHLF,STEP,TM,XI
A=XP*VEL(4)
A=A-VEL(5)
V=-A#2*VEL(3)*VEL(4)
A=A*A
A=VEL(3)*A
A=VEL(2)*EXP(-A)
DV=V*A
V=VEL(1)+A
RETURN
END

```

```

CRUSS001
CRUSS002
CRUSS003
CRUSS004
CRUSS005
CRUSS006
CRUSS007
CRUSS008
CRUSS009
CRUSS010
CRUSS011
CRUSS012
CRUSS013
CRUSS014
CRUSS015
CRUSS016
CRUSS017
CRUSS018
CRUSS019
CRUSS020
CRUSS021
CRUSS022
CRUSS023

```

```

SURROUTINE FCT
DOUBLE PRECISION DERY,Y,PKMT,X,TH,AP,YP,U,B,I,M,BX,S,K,XLIM,AUX,XX
DOUBLE PRECISION A,C,LCOS,DSIN,DELX
REAL I1,I2,I3,I4,I5,I6,I7,I8,M
INTEGER P,RR,REG,VFG,FLAG
DIMENSION PRMT(5),Y(10),AUX(8,10),DERY(10),A(10,10),C(10),VEL(0)
EQUIVALENCE (Y(3),TH),(Y(7),XP),(Y(10),YP),(Y(3),B),(Y(1),U)
EQUIVALENCE (Y(2),T),(Y(4),H),(Y(7),BX),(Y(6),S),(Y(5),R)
COMMON DERY,Y,PKMT,X,XLIM,AUX,XX,A,C,DELX
COMMON FPSB,G,DV,IER,IR,WI,VI,RMI,M,FR,AS,E
COMMON THID,THI,ERR,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,DT,FI
COMMON P,RR,NPAGE,NLINE,REG,NDIM,V-G,FLAG,IHLF,STEP,IM,XI
DO 5C I=1,10
C(I)=0.
DERY(I)=0.
DO 50 JI=1,10
A(JI,I)=0.
50 CONTINUE
101 FLAG=VFG
WI=EPS#U
VI=-WI
EPSB=EPS
ARG=5*TH/(U*U*FR*FR)
ARG=ARG/(AS**5)
IF (ARG)10100,10100,10101
10100 ARG=C.
10101 IF (ARG-100.)10103,10103,10102
10102 WIR=C.
GO TO 10104
10103 WIR=EXP(-ARG)
10104 GO TO (1011,1012), FLAG
1011 G=0.
BXS=BX
BX=0.
GO TC 1013
1012 C=FI#T

```

FCTN0001
 FCTN0002
 FCTN0003
 FCTN0004
 FCTN0005
 FCTN0006
 FCTN0007
 FCTN0008
 FCTN0009
 FCTN0010
 FCTN0011
 FCTN0012
 FCTN0013
 FCTN0014
 FCTN0015
 FCTN0016
 FCTN0017
 FCTN0018
 FCTN0019
 FCTN0020
 FCTN0021
 FCTN0022
 FCTN0023
 FCTN0024
 FCTN0025
 FCTN0026
 FCTN0027
 FCTN0028
 FCTN0029
 FCTN0030
 FCTN0031
 FCTN0032
 FCTN0033
 FCTN0034
 FCTN0035
 FCTN0036

1013 WI=WI*WIR
 102 GO TO (102,103,104,105), REG
 102 WI=WI*(I1-I2)
 VI=VI*(I1-I2)
 GO TO 110
 103 WI=WI*(I1-I2)
 VI=VI*I1/2
 GO TO 110
 104 WI=WI*I1/2
 VI=VI*(I1-I2)
 GO TO 110
 105 WI=WI*I1/2
 VI=VI*I1/2
 110 H1=R+H*I1
 B1=S+B*I1
 A(1,1)=H1*B1
 A(1,2)=U*I1*B1
 A(1,3)=U*B1
 A(1,4)=U*H1
 C(1)=WI*B1-VI*H1-U*I1*H1*B1X
 F3=R+H*I3
 B3=S+B*I3
 H2=R+H*I2
 B2=S+B*I2
 HG=R*R/2+R*H*I3+H*H*I4
 A(2,1)=2*U*H2*B2
 A(2,2)=U*U*I2*B2+G*B3*(R*I3+2*H*I4)
 A(2,3)=U*U*B2+G*B3*(R+H*I3)
 A(2,4)=U*U*H2+G*HG
 A(2,5)=F1*B3*HG
 C(2)=- (U*U*I2*H2+G*I3*HG)*B3X
 B7=S+B*I7
 H7=R+H*I7
 A(3,1)=G*B7*H7
 A(3,2)=U*G*I7*B7
 A(3,3)=U*G*B7

FCIN0037
 FCIN0038
 FCIN0039
 FCIN0040
 FCIN0041
 FCIN0042
 FCIN0043
 FCIN0044
 FCIN0045
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 FCIN0090
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 FCIN0095
 FCIN0096
 FCIN0097
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 FCIN0099
 FCIN0100
 FCIN0101
 FCIN0102
 FCIN0103
 FCIN0104
 FCIN0105
 FCIN0106
 FCIN0107
 FCIN0108

```

A(3,4)=U*G*H7
A(3,5)=U*H7*F7*FI
C(3)=-E*G*H3-U*G*1/*H7*B X
GU TC (1,2,3,4), REG
1 A(5,1)=1
  A(5,2)=G*I3
  A(5,3)=G
  A(5,5)=FI*(R/2+H*I3)
  GO TC 5
2 A(5,4)=1.
  GO TC 5
3 A(5,3)=1.0
  GO TC 5
4 A(5,4)=1.
  A(4,3)=1.0
  GO TC 6
5 I3=1.-I2/I1
  A(4,1)=U*((S*H+R*B)*I2+R*S*I3)
  A(4,2)=G*(2*S*H*I4+R*E*I3*I5)
  A(4,3)=U*U*S*I8+G*(S*F*I3+R*B*I3)
  A(4,4)=U*U*R*I8+G*(R*R/2+H*R*I3)
  A(4,5)=FI*(S*H*H*I4+I*R*R*I4/2+B*H*R*I3*I3)
  C(4)=U*(VI*R-WI*S)*I2/I1-G*(P*R/2+H*R*I3)*I3*B X
  GO TC (120,131), VFG
120 CALL CROSS
  DERY(9)=DSIN(TH)
  DERY(10)=CCOS(TH)
  VST=V*DERY(9)
  VCT=V*DERY(10)
  DT=U*U*(S+B*I2)*(R+H*I2)+2*J*VCT*(S+B*I1)*(R+H*I1)
  1+VCT*VCT*(R+H)*(S+B)
  CT=-VST*(WI*(S+B*I1)-VI*(R+H*I1))/DT
  DERY(8)=UT
  FO=R+H
  DT=VST*DT-DERY(10)*DV*DERY(9)
  EO=S+B
  
```

```

A(1,2)=A(1,2)+VCT*B3
A(1,3)=A(1,3)+VCT*B3
A(1,4)=A(1,4)+VCT*B3
C(1)=C(1)-VCT*H3+BC*H3*H3
A(2,1)=A(2,1)+2*VCT*H1*B1
A(2,2)=A(2,2)+VCT*(2*U*11*B1+VCT*B3)
A(2,3)=A(2,3)+VCT*(2*U*11*B1+VCT*B3)
A(2,4)=A(2,4)+VCT*(2*U*11*B1+VCT*B3)
C(2)=C(2)-VCT*(2*U*11*B1+VCT*B3)+2*(U*H1+VCT*B3)*H3*H3
1+VCT*(H1*B1-VI*H1)
A(3,2)=A(3,2)+G*VCT*I2*B3
A(3,3)=A(3,3)+G*VCT*B2
A(3,4)=A(3,4)+G*VCT*B2
A(3,5)=A(3,5)+F1*VCT*B3*B3
C(3)=C(3)-G*VCT*I3*B3+H3*H3*H3*H3*H3*H3
GO TC (7,6,3,5),RFG
7 A(5,1)=A(5,1)+VCT
C(5)= (U+VCT)*DT
GO TC 9
8 A(4,1)=A(4,1)+VCT*I1*(R*B+S*H)
A(4,2)=A(4,2)+S*U*VCT*(11-11/12)
A(4,3)=A(4,3)+S*U*VCT*I8
A(4,4)=A(4,4)+R*H1*VCT*I8
C(4)=C(4)-R*U*VCT*(11-11/12)*B3
1+(U*(2*I1-12/11)+VCT)*(S*H+R*B)+R*S*U*I8)*DT
9 GO TC (130,131),FLAG
130 A(1,2)=A(1,2)+(U*11*B1+VCT*B3)
A(2,2)=A(2,2)+(U*11*B1+VCT*B3)+2*U*VCT*(11*B1+VCT*B3)*H3
GO TC (10,10,10,11),REG
10 A(4,2)=A(4,2)+R*U*VCT*(11-12/11)
11 CALL SGFLG(5)
EPSB=C(2)
EX=B3
FLAG=2
C=FI*H
WI=WI*WI*H

```

FCIN0109
 FCIN0110
 FCIN0111
 FCIN0112
 FCIN0113
 FCIN0114
 FCIN0115
 FCIN0116
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 FCIN0136
 FCIN0137
 FCIN0138
 FCIN0139
 FCIN0140
 FCIN0141
 FCIN0142
 FCIN0143
 FCIN0144

```

GO TO 110
131 DB=BX-EP58
  A(6,1)=DB*2*U*B#H2*I6
  A(6,2)=DB*U*U*B#I2*I6
  A(6,3)=DB*U*U*B#I6
  A(6,6)=U*U*B#H2*I6
  C(6)=G*HG-U*U*I6*DB*F2*BX
GO TO (132,133),VFG
132 A(6,1)=A(6,1)+2*B*VCT*I5*H2*DB
  A(6,2)=A(6,2)+VCT*B*(2*U*I5*I1+VCT)*DB
  A(6,3)=A(6,3)+VCT*B*(2*U*I5+VCT)*DB
  A(6,6)=A(6,6)+VCT*B*(2*U*I5*H1+VCT*H3)
  C(6)=C(6)-DB*VCT*(2*U*I5*H1+VCT*H3)*BX+2*DB*B*(U*I5*H1+VCT
1*H3)*DT
133 CALL SGELG(5)
  DERY(1)=C(1)
  CERY(2)=C(5)
  CERY(3)=BX
  DERY(4)=C(2)
  CERY(5)=C(3)
  DERY(6)=C(4)
  CERY(7)=C(6)
  RETURN
  END

```

```

FCIN0145
FCIN0146
FCIN0147
FCIN0148
FCIN0149
FCIN0150
FCIN0151
FCIN0152
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FCIN0154
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FCIN0156
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FCIN0162
FCIN0163
FCIN0164
FCIN0165
FCIN0166
FCIN0167
FCIN0168

```

```

SUBROUTINE SCERG(M)
DOUBLE PRECISION AD, AI, XX, DABS, DCOS, DSIN, SAVE, S, R, XLIM, AUX
DOUBLE PRECISION DERY, Y, PRMT, X, TH, XP, YP, L, B, T, H, BX
DOUBLE PRECISION A, PIV, TB, TOL, PIVI, DELX
REAL MUN
INTEGER P, RR, REC, VFG, FLAG
DIMENSION PRMT(5), Y(10), AUX(5, 10), DERY(10), AI(100), R(10), VEL(8)
DIMENSION A(100), AD(10, 10)
DIMENSION SAVE(10)
EQUIVALENCE (AI(1), AD(1, 1))
COMMON DERY, Y, PRMT, X, XLIM, AUX, XX, AI, R, DELX
COMMON EPS3, G, DV, IER, VIK, VI, VJ, RMI, MUN, FR, AS, E
COMMON THID, TH, THERK, VEL, IL, I2, I3, I4, I5, I6, I7, I8, EPS, V, BXS, DT, FI
COMMON P, RR, NPAGE, NLINE, REG, NDIM, VFG, FLAG, IHLF, STEP, IM, XI
A=1
EPSS=.1E-16
SCALE THE MATRIX
DO 54 I=1, M
  AMAX=R(I)
  DO 52 J=1, M
    DA=DABS(AD(I, J))
    IF (DA-AMAX) 52, 52, 51
51 AMAX=DA
52 CONTINUE
  DO 53 J=1, M
    AD(I, J)=AD(I, J)/AMAX
53 CONTINUE
  R(I)=R(I)/AMAX
54 CONTINUE
  SAVE THE Y EQUATION ROW
  MM=M+1
  DO 999 I=1, MM
    SAVE(I)=AD(MM, I)
999 CONTINUE
  CONVERT MATRIX TO COLUMN FORM
  IJ=0

```

SGFL0001
 SGFL0002
 SGFL0003
 SGFL0004
 SGFL0005
 SGFL0006
 SGFL0007
 SGFL0008
 SGFL0009
 SGFL0010
 SGFL0011
 SGFL0012
 SGFL0013
 SGFL0014
 SGFL0015
 SGFL0016
 SGFL0017
 SGFL0018
 SGFL0019
 SGFL0020
 SGFL0021
 SGFL0022
 SGFL0023
 SGFL0024
 SGFL0025
 SGFL0026
 SGFL0027
 SGFL0028
 SGFL0029
 SGFL0030
 SGFL0031
 SGFL0032
 SGFL0033
 SGFL0034
 SGFL0035
 SGFL0036

```

AM=0
DO 1100 K=1,M
CO 1000 L=1,M
IJ=IJ+1
NM=NM+1
1000 A(IJ)=AI(NM)
1100 NM=NM+10-M
IF(M)23,23,1
C
C SEARCH FOR GREATEST ELEMENT IN MATRIX A
1 IER=0
PIV=C.00
MM=M*M
AM=N*M
DO 3 L=1,MM
TB=DABS(A(L))
IF(TB-PIV)3,3,2
2 PIV=TB
I=L
3 CONTINUE
TOL=EPSS*PIV
A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I)
C
C
C
C START ELIMINATION LUCL
LST=1
CO 17 K=1,M
C
C TEST ON SINGULARITY
IF(PIV)23,23,4
4 IF(IER)7,5,7
5 IF(PIV-TCL)6,6,7
6 IER=K-1
7 PIV=1.00/A(I)
J=(I-1)/M
I=I-J*M-K

```

SGEL0037
 SGEL0038
 SGEL0039
 SGEL0040
 SGEL0041
 SGEL0042
 SGEL0043
 SGEL0044
 SGEL0045
 SGEL0046
 SGEL0047
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 SGEL0049
 SGEL0050
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 SGEL0064
 SGEL0065
 SGEL0066
 SGEL0067
 SGEL0068
 SGEL0069
 SGEL0070
 SGEL0071
 SGEL0072

```

C
C
C
J=J+1-K
I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT
PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R
DO 8 L=K,NM,M
LL=L+1
TB=PIVOT*A(LL)
A(LL)=R(L)
3 R(L)=TB
C
C
C
IS ELIMINATION TERMINATED
IF (K-M) 9,18,18
C
C
C
COLUMN INTERCHANGE IN MATRIX A
9 LEND=LST+M-K
IF (J) 12,12,10
10 II=J*M
DO 11 L=LST,LEND
TB=A(L)
LL=L+1
A(L)=A(LL)
11 A(LL)=TB
C
C
C
ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A
12 DO 13 L=LST,M,M
LL=L+1
TB=PIVOT*A(LL)
A(LL)=A(L)
13 A(L)=TB
C
C
C
SAVE COLUMN INTERCHANGE INFORMATION
A(LST)=J
C
C
C
ELEMENT REDUCTION AND NEXT PIVOT SEARCH
PIV=0.00
LST=LST+1

```

SGEL0073
 SGEL0074
 SGEL0075
 SGEL0076
 SGEL0077
 SGEL0078
 SGEL0079
 SGEL0080
 SGEL0081
 SGEL0082
 SGEL0083
 SGEL0084
 SGEL0085
 SGEL0086
 SGEL0087
 SGEL0088
 SGEL0089
 SGEL0090
 SGEL0091
 SGEL0092
 SGEL0093
 SGEL0094
 SGEL0095
 SGEL0096
 SGEL0097
 SGEL0098
 SGEL0099
 SGEL0100
 SGEL0101
 SGEL0102
 SGEL0103
 SGEL0104
 SGEL0105
 SGEL0106
 SGEL0107
 SGEL0108


```

R(J)=R(K)
21 R(K)=TB
22 GO TO (223,221), FLAG
221 M=M+1
    DO 222 I=1,M
        R(M)=R(M)-SAVE(I)*R(I)
222 CONTINUE
        R(M)=R(M)/SAVE(M)
223 DO 24 I=1,LOC
        A(I)=0.
24 CONTINUE
    RETURN
C
C
C      ERROR RETURN
23 IER=-1
    RETURN
    END

```

```

SGFL0149
SGFL0150
SGFL0151
SGFL0152
SGFL0153
SGFL0154
SGFL0155
SGFL0156
SGFL0157
SGFL0158
SGFL0159
SGFL0160
SGFL0161
SGFL0162

```

Input:

6.0
0.0

0.6
0.0

0.0
0.0

500.0
0.0

0.01
0.0

1.0
0.0

Output:

BUCHANAN JET CALCULATIONS

PAGE 1

PROJEC NUMBER
6.00000

ASPECT RATIO
0.60300

ANGLE OF DISCHARGE
90.00000

ROLL-OFF ERROR
0.01000

XLIM
500.00000

HEAT LOSS CORR
0.0

VEL1=	0.0	VEL2=	0.0	VEL3=	0.0	VEL4=	0.0	VEL5=	0.0	VEL6=	0.0	VEL7=	0.0	VEL8=	0.0
X	H	B	R	S	FL	Q	M	U	T	HT	V	XP	VP	THQ	TM
0	1000-03	1000-03	7750 C0	1290 C0	528E 03	1.00E 01	1.014	1.000	1.000	1.000	0.0	0	0	90.0	0
1310 C1	1900 00	5850 00	5210 00	1520 C0	124E 02	1.09E 01	1.014	1.005	0.571	1.000	0.0	1310 01	0	90.0	131F 01
2620 01	3780 00	1160 01	4500 00	1370 C1	890E 01	1.18E 01	1.014	1.004	0.540	1.000	0.0	2620 01	0	90.0	262F 01
5240 C1	7240 00	2390 01	2180 00	1440 C1	664E 01	1.13E 01	1.014	1.006	0.584	1.000	0.0	5240 01	0	90.0	524E 01
7660 C1	1040 01	3770 01	2930-C1	1530 C1	574E 01	1.161E 01	1.014	1.007	0.828	1.000	0.0	7660 01	0	90.0	766E 01
9480 01	1190 01	6500 01	0	1520 C1	517E 01	1.177E 01	1.014	1.007	0.759	0.986	0.0	9480 01	0	90.0	948E 01
1150 02	1340 01	5270 01	0	1480 C1	465E 01	1.196E 01	1.000	0.997	0.692	0.986	0.0	1150 02	0	90.0	115E 02
1310 02	1610 01	6520 01	0	1300 C1	391E 01	1.232E 01	1.000	0.997	0.597	0.986	0.0	1310 02	0	90.0	131E 02
1570 02	1850 01	8730 01	0	1000 C1	342E 01	1.265E 01	1.000	0.997	0.535	0.990	0.0	1570 02	0	90.0	157E 02
2100 02	2210 01	1250 02	0	1950 C0	283E 01	1.321E 01	1.002	0.999	0.457	0.990	0.0	2100 02	0	90.0	210E 02
2360 02	2240 01	1530 02	0	1030 C0	267E 01	1.347E 01	1.000	0.999	0.424	0.987	0.0	2360 02	0	90.0	236E 02
2490 02	2270 01	1660 02	0	0	258E 01	1.360E 01	1.000	0.999	0.409	0.985	0.0	2490 02	0	90.0	249E 02
2750 02	2280 01	1960 02	0	0	250E 01	1.373E 01	1.000	0.999	0.394	0.985	0.0	2750 02	0	90.0	275E 02
2880 02	2270 01	2110 02	0	0	244E 01	1.387E 01	1.000	0.999	0.381	0.985	0.0	2880 02	0	90.0	288E 02
3150 02	2250 01	2460 02	0	0	238E 01	1.400E 01	1.000	0.999	0.368	0.985	0.0	3150 02	0	90.0	315E 02
3670 02	2200 01	3240 02	0	0	227E 01	1.426E 01	1.000	0.999	0.346	0.985	0.0	3670 02	0	90.0	367E 02
4190 02	2150 01	4150 02	0	0	211E 01	1.476E 01	1.000	0.999	0.310	0.985	0.0	4190 02	0	90.0	419E 02
4720 02	2090 01	5160 02	0	0	197E 01	1.523E 01	1.000	0.999	0.282	0.985	0.0	4720 02	0	90.0	472E 02
5240 02	2040 01	6250 02	0	0	186E 01	1.568E 01	1.000	0.999	0.260	0.985	0.0	5240 02	0	90.0	524E 02
6290 02	2020 01	8840 02	0	0	177E 01	1.604E 01	1.000	0.999	0.242	0.985	0.0	6290 02	0	90.0	629E 02
7340 02	2110 01	1240 03	0	0	151E 01	1.683E 01	1.000	0.999	0.215	0.984	0.0	7340 02	0	90.0	734E 02
8390 02	1950 01	1740 03	0	0	107E 01	1.707E 01	1.000	0.999	0.208	0.984	0.0	8390 02	0	90.0	839E 02
1050 03	1560 01	3240 03	0	0	868E 00	1.715E 01	1.000	0.999	0.206	0.983	0.0	1050 03	0	90.0	105E 03
1260 03	1250 01	5520 03	0	0	658E 00	1.720E 01	1.000	0.999	0.202	0.978	0.0	1260 03	0	90.0	126E 03
1470 03	1020 01	8640 03	0	0	543E 00	1.723E 01	1.000	0.999	0.202	0.974	0.0	1470 03	0	90.0	147E 03
1680 03	3590 00	1260 04	0	0	470E 00	1.723E 01	1.000	0.999	0.202	0.971	0.0	1680 03	0	90.0	168E 03
2100 03	8410 00	2350 04	0	0	349E 00	1.721E 01	1.000	0.999	0.200	0.963	0.0	2100 03	0	90.0	210E 03
2520 03	5060 00	3820 04	0	0	304E 00	1.716E 01	1.000	0.999	0.200	0.956	0.0	2520 03	0	90.0	252E 03

JET VELOCITY HAS BECOME TOO SMALL RUN TERMINATES

Appendix III Input and Output